## IX.-WORLD'S INDEX-NDMBER OF PRICES.

1. General.-Very diverse reasons have been put forward for the world-wide movements in prices, such as the rise which occurred prior to the early seventies, the following decline, and, again, the rise in more recent years. While it is not the essential aim of this Report to analyse the cause of price movements, it is desirable to refer briefly to certain statistical and historical aspects of what is so frequently stated to constitute one of the main controlling factors-viz., the world's goldsupply.

The following table shews the value of the average annual gold production of the world during each year since 1851, and the estimated annual production during the period 1840 to 1850 . The "world's index-number of prices," shewn in the same table, has been compiled from the index-numbers for the countries already referred to, by weighting each index-number by a number proportional to the population of the country to which it refers (see p. 76 hereinbefore).

World's Index Numbers and World's Gold Production, 1840 to 1911.

| Year. | $\begin{aligned} & \text { World's Index } \\ & \text { No.* } \end{aligned}$ | World's Gold Production. $£ 0,000$ omitted. | Year. | $\underset{\substack{\text { World's Index } \\ \text { No.* }}}{\text { Nos. }}$ | Workl's Gold Produtction. £0,000 onitted. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | - |
| 1840 | 1,165 | ) | 1876 | . 1,056 | 2,074 |
| 1841 | 1,146 | , | 1877 | 1,030 | 2,280 |
| 1842 | 1,058 |  | 1878 | 950 | 2,380 |
| 1843 | 985 |  | 1879 | 903 | 2,180 |
| 1844 | 992 |  | 1880 | 1,016 | 2,132 |
| 1845 | 1,010 | 1,250 | 1881 | 955 | 2,062 |
| 1846 | 1,041 |  | 1882 | 964 | 2,040 |
| 1847 | 1,052 |  | 1883 | 938 | 1,908 |
| 1848 | 961 |  | 1884 | 887 | 2,034 |
| 1849 | 956 |  | 1885 | 838 | 2,168 |
| 1850 | 996 | ) | 1886 | 807 | 2,120 |
| 1851 | 928 | 1,354 | 1887 | 809 | 2,106 |
| 1852 | 923 | 2,656 | 1888 | 837 | 2,204 |
| 1853 | 1.023 | 3,110 | 1889 | 853 | 2,470 |
| 1854 | 1,071 | 2,550 | 1890 | 865 | 2,377 |
| 1855 | 1,073 | 2,702 | 1891 | 855 | 2,613 |
| 1856 | 1,095 | 2,952 | 1892 | 825 | 2,926 |
| 1857 | 1,115 | 2,666 | 1893 | 810 | 3,169 |
| 1858 | , 091 | 2,494 | 1894 | 750 | 3,650 |
| 1859 | 1,004 | 2,498 | 1895 | 732 | 3,980 |
| 1860 | 1,026 | 2,386 | 1896 | 716 | 4,225 |
| 1861 | 1,033 | 2,276 | 1897 | 721 | 4,820 |
| 1862 | 1,102 | 2,156 | 1898 | 749 | 5,814 |
| 1863 | 1,243 | 2,140 | 1899 | 797 | 6,301 |
| 1864 | 1,416 | 2,260 | 1900 | 864 | 5,209 |
| 1865 | 1,463 | 2,404 | 1901 | 831 | - 5,334 |
| 1866 | 1,368 | 2,420 | 1902 | 830 | 6,062 |
| 1867 | 1,273 | 2,080 | 1903 | 847 | 6,676 |
| 1868 | 1,212 | 2,194 | 1904 | 850 | 7,052 |
| 1869 | 1,186 | 2,124 | 1905 | 864 | 7,688 |
| 1870 | 1,137 | 2,138 | 1906 | 923 | 8,317 |
| 1871 | 1,142 | 2,140 | 1907 | 978 | 8,476 |
| 1872 | 1,206 | 1,992 | 1908 | 915 | 9,036 |
| 1873 | 1,215 | 1,924 | 1909 | 931 | 9,308 |
| 1874 | 1,153 | 1,816 | 1910 | -970 | 9,419 |
| 1875 | 1,100 | 1,950 | 1911 | 1,000 | . |

[^0]The preceding figures are shewn in the following graph:-
WORLD'S INDEX-NUMEERS AND WORLD'S GOLD-SUPPLY, 1840 TO 1911.


The increase in the annual gold production and the rise and fall in average prices may be more readily seen by taking averages for quinquennial periods. The following table accordingly shews the value of the average annual gold production and the average of the annual indexnumbers for each quinquennial period since 1841 :-

Value of Average Annual Worid's Gold Production and Average of corresponding Worlds' Index-Number for each Quinquennial Period from 1841 to 1910.

| Period. | Average <br> Index-Number. | Average Annual Gold Production. ( $\mathbf{£} 0,000$ ) | Perlod. | Average Index-Number. | Average Amnusl Gold Production. ( $\mathbf{( 1 0 , 0 0 0 ) .}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1841-55 | 1,088 |  | 1881-85 | 916 | 2,042 |
| 1846-50 | 1,001 | $\} \quad 1,250$ | - 1886-90 | 834 | 2,255 |
| 1851-55 | 1,003 | 2,474 | - 1891-95 | 794 | 3,268 |
| 1856-60 | 1,046 | 2,509 | 1896-1900 | 769 | 5,274 |
| 1861-65 | 1,251 | 2,247 | 1901.05 | 844 | 8,562 |
| 1866-70, | 1,235 | 2,101 | 1906-10 | 943 | 8,911 |
| 1871-75' | 1,163 | 1,964 | 1911 | 1,000 | - |
| 1876-80 | 991 | 2,209 |  |  |  |

* Not available.

The average value of the world's production during the decade 1841 to 1850 was only $£ 12,500,000$; but it may be seen that in the next quinquennium the average value of the production had risen to $£ 24,740,000$ and $£ 25,990,000$ between 1856 and 1860, viz., during the great impetus given by the almost simultaneous discoveries in Australia and California. It then fell to $£ 19,640,000$ in 1871-5.

When the rich alluvial deposits in these countries began to be worked out, and until the opening of the Transvaal mines, it remained fairly constant. It was then, viz., in about 1888, that the production commences to take an upward sweep, as the graph will shew. In 1891 the value produced exceeded $£ 26,000,000$, in 1898 it was $£ 58,140,000$, and in $1899 £ 63,010,000$; then after a momentary reaction, caused by the South African War, the output rose to $£ 66,760,000$ in 1903 , and to $\mathfrak{£} 94,190,000$ in 1910. From 1896 to 1910 the output increased rapidly. The opening of the Klondike mines and the discovery and application of the cyanide process had important effects on the production. Thus, in 1910 the output was very nearly four times as great as in 1890, twenty years earlier.

The index-numbers given in the above table, being based on prices in all countries for which index-numbers are available, are, of course, more directly applicable than any others in an investigation into the relation between world's gold-supply and prices.

Now an examination of these two graphs shews several instances of corresponding upward or downward movements, which may be to some extent due to some law acting between the gold-supply and the world's prices, though, even if this be so, prices do not, in general, seem to feel the reaction set up by a change in the gold-supply, at least not till after a very noticeable interval. It is probably true that the important factor to consider, when estimating the effect of gold on prices, is not so much the annual production, as the quantity and rapidity of gold in circulation relatively to the varying demand for gold, arising, among other things, by extensions or contractions of credit. Notwithatanding that a large quantity of statistical material bearing more or less directly on these matters has been collected, no reliable estimates of gold in circulation, and therefore of velocity of circulation, are available, except for one or two countries.
2. From 1850 to $\mathbf{1 8 5 \%}$.-One of the most marked and frequently cited coincidences between increase in gold-production and a rise in prices is that which marks the periods between 1851 and 1857, following the abnormal and exceptional discoveries in Australia and California.

The increased output of gold in the fifties first found its way to Europe and the United States, and resulted in a large increase in the coinage of gold in England, France, and the United States, and thus, in the quantity of gold in actual circulation. The impetus given to general settlement by the gold discoveries (as in Victoria) created a demand for manufactured commodities, which tended to accelerate the rise in prices. Other influences, too; were operating in the same direction. The Crimean War tended to raise the price of many commodities, while in various countries, and especially in England, it is probable that a considerable extension of credit took place. Jnder the stimulus of abundant loan capital and an optimistic spirit of expanding trade, many new enterprises were started. During the years 1853 to 1856 there were bad seasons in England, and the price of foodstuffs rose to a high point. In 1857-8-at the time of the Indian Mutiny-the index-number shews a decline from 1115 to 991 , and then, again, moves upward until 1865, when it was as high as 1463.
3. Prices High from 1858 to $18 \% 3$. -The expansion of trade in the sixties was greatly assisted by the extension of the Limited Liability Acts to banking corporations in 1858, and by the passing of the Company Act, 1861, which gave a great impetus to the flotation of jointstock enterprises. Inventions and discoveries also did much to assist the boom, as also did improvements in transport facilities and in the arts of manufacture. The introduction of the Bessemer process of steel making in 1859 is a landmark in the industrial history of the period.

Important influences are, on the other hand, commonly supposed to have operated in the opposite direction, such as the numerous wars during the fifties and sixties, which not merely kept men from productive occupations, but caused a considerable loss in life and property. When it is remembered that the period includes, among others, the Crimean, Austro-Prussian, Franco-Prussian, Danish and Italian wars in Europe, and the Civil War and the Mexican campaigns of Napoleon III. in America, it may readily be appreciated that war must have had adverse effects on trade and general prosperity.

From 1858 to 1865 prices rose rapidly, largely owing to the abnormal conditions in the United States of America. During this period, however, the gold production generally decreased. Between 1862 and 1866, the American Civil War cut off Lancashire's supply of cotton, and though this shortage stimulated cotton-growing in India and other countries, the supplies were quite insufficient. The cotton famine naturally had an effect on other textiles, and the price of wool and flax rose rapidly. After the end of the war prices again fell, but not quite to their old level. In the following period up to 1873, the output of gold continued to decrease, while prices shewed a marked rise from 1871 to 1873. Reviewing the whole period from 1855 to 1875 , the decrease in the gold-production, coinciding with an extension of civilisation and trade over the whole world, and with new needs for gold, ought, it seems, on the theory which holds that the supply of gold is the supreme influence, to have led to a general fall in the price of commoditios. Still, it. is contended that the increased gold output of the previous years had not yet become insufficient to meet the consumption at the enhanced prices, and so the continuation of industrial development would still tend to exereise a predominant influence. So that, although it is stated that in the case of many manufactured articles a fall in price really occurred, owing to the boom in trade, reduced cost of transport, and other causes, natural products continued to increase in price, and the indexnumber for all commodities to rise. Speaking generally, it is probably true that the annual production of gold during the twenty-two years, from about 1851 to 1873 , was available for monetary purposes, whereas the quantity available for the following two decades was largely absorbed by the extra demand due to changes in the monetary systems, so that the total monetary circulation did not increase during the latter period, even relatively to the reduced output of gold, to the same extent as in the former period.
4. Prices Falling from 1873 to 1896. -The period 1873 to 1896 was marked by a progressive fall in prices, this time probably in conformity with the stagnation or recoil in the supply of gold in relation to the demand therefor. It may be seen (see page 87) that from 1878 to 1883 , there was a considerable decline in the world's production
of gold, but that after the following six years the output regained its former value. The important fact appears, however, to be that these years saw a great increase in the demand for gold. Immediately after the Franco-Prussian war, Germany decided to establish her currency on a gold basis, and a law to that effect was passed in December, 1871. The gold standard was not introduced until 1873, though it is slated that considerable importations of the precious metal took place immediately.* Further, the United States commenced to draw gold from Europe in 1878, consequent upon a law making the inconvertible Government bank-notes, which had been issued during the Civil War, convertible into gold at the United States Treasury. There is no reason to doubt that this resumption was followed by a great extension in the use of gold, and a country which was formerly one of the chief sources of supply began to reabsorb some of the world's gold. In other countrics, too, currency reforms were effected at or about this period, and gold became practically the sole standard measure of value.* It may here be observed that the question of currency reforms in relation to the supply of gold is greatly complicated by various considerations, such as (a) changes in the rapidity of circulation of money, (b) cconomies in the use of coin either in international trade, by the payment of balances, by the transfer of stocks, or in general trade by the extension of banking facilities on the introduction of paper money, and (c) the amount of credit instruments in circulation, which varies with the commercial habits of the people and the character of the barking system.

It is alleged by many economists that the increase in production at this period had an important effect on prices. The highly remunerative prices hitherto prevailing are stated to have greatly stimulated production, and when prices declined, it is said that producers in many branches of trade were obliged to further increase their production, in order to balance, at least to some extent, the shrinkage of values.

The diminution in the cost of production and conveyance at this time probably had its greatest effect in extra-European Countries. As there had been a European era of the development of steam and of railways, so there was now an extra-European era. Not only were existing settlements connected, but the railroads were taken through uncultivated and sparsely populated districts, which were thus opened for new settlement. The producer could not only convey his products cheaper and quicker to port and to other countries, but he could also obtain his requirements at less cost and more rapidly. The effect of Jong distances was reduced owing to the better communications, and the increasing civilisation in the new countries attracted an increasing number of immigrants. Land was abundant, and other factors of production, such as labour and capital, were more easily acquired.

Steamers superseded more and more sailing vessels; their number increased enormously, and as they travelled three times as quick, their tonnage counted thrice that of the sailers. The opening of the Suez Canal had an additional influence in accelerating the conveyance of goods.

[^1]Another influence upon commerce generally may be ascribed to telegraphic communication; this kept producer and consumer in closer touch. As products could be quickly exchanged between various countries, less dependence was placed upon stocks, and the market could not be so easily manipulated by speculators. Also, the effect of bad harvests in certain districts or countries was minimised, as, thanks to the quick communications, a deficit in one country could be covered by an excess in another. If heavy stocks still existed, it was owing chiefly to the great production, and the unprofitable state of business, not to any great necessity for them; they consequently weighed upon the market with additional force, and caused a greater depression than similar quantities would have occasioned in former periods.

Last, but not least, may be mentioned the inventions of the period. Attention may specially be directed to the improvenents in the smelting of ore, in the production of steel, and in the sugar industry; to the development of the chemical industry, and to the improvement of machinery in all branches.

To sum up, the following are the causes which have been alleged to be responsible for the appreciation in the purchasing power of gold during the period between 1873 and 1886.*
(a) Reduction in the cost of production would tend to cause the prices of these commodities to fall and to produce changes in relative prices, but would have no effect on the general price-level unless the quantities produced were increased.
(b) Reduction in the cost of transport would produce no effect on the general price-level, unless it led to an increase in the quantities of commodities produced, or to an increase in the number of exchanges.
(c) The reduced cost of production and the reduction of cost of transport would probably, and did in fact, cause changes in the relative advantages of different countries in the international trade of the world, which would have the effect of altering the internal scale of prices and wages in the countries affected.
(d) There was an increase in the quantities of commodities produced and an increase in the number of exchanges, both causes tending to bring about a fall in the general price-level.
(e) There were additional demands for gold due to the substitution of the gold for the silver standard in certain countries.
( $f$ ) There were additional demands for gold due to changes from inconvertible paper to a metallic (gold) standard.
(g) There were special demands for gold due to the great development of the United States of America. This cause is, to some extent; identical with that stated in (c).
( $h$ ) There was some reduction in the yearly production of gold.
From 1890 onwards the gold output began to rise with a rapidity probably without precedent. Yet the fall in prices for some years steadily continued, with a few interruptions (from 1886 to 1890) until 1896, thus shewing once more the same discordance, at least apparently, between the two phenomena. Perhaps there is something

[^2]analogous at this stage to what occurred in the other direction during the period between 1858 and 1873 ; and it is probable that any influence which the output of gold exercises on prices takes some years to shew itself. In any case, moreover, evidence as to increase in gold supply does not appear to be conclusive, until it is known whether a greater increase in the number of business transactions occurred involving increased use for gold. A feature of the period was the increase in overseas trade. It is obvious that increase of trade of this kind tends to bring about an even greater use of currency than increase in domestic trade, for every time that a commodity changes bands metallic currency or a credit document of some kind is given in exchange, and commodities brought to market from overseas will ordinarily change hands a greater number of times than domestic produce. Thus, as trade is'developed and becomes more world-wide, a greater demand for currency or its equivalent tends to be brought about. It is unlikely, indeed, that there was any actual shortage of gold during this period, but prices are determined not only by the absolute amount of currency, but by the relative quantity of currency as compared with the volume of trade which it has to do and with other matters.

This view of the question has been presented by Professor Irving Fisher,* who points out that the total amount of money expended on commodities in a given community during a given period is equal to sum of the product of the average price of sale of each commodity into the quantity of such commodity sold. This must be equal to the amount of money in circulation among the community for that period, multiplied by its velocity of circulation. The money by which payment is made consists of (a) actual coin in circulation, and (b) credit money based on gold and on other forms of property deposited in the banks, the latter usually taking the form of bank-notes, cheques, and bills of exchange. $\dagger$ Therefore, it follows that the sum of (a) the amount of legal-tender currency in circulation multiplied by its velocity of circulation, and (b) the amount of credit money in circulation multiplied by its velocity, is equal to the sum of the prices of all commodities multiplied respectively by the quantity of each commodity sold. The general price-level, or the average of the prices, for the period is therefore equal to the sum of (a) the legal-tender currency multiplied by its velocity of circulation, and (b) the credit money multiplied by its velocity, divided by the total quantity of commodities sold. $\ddagger$ It is, therefore, clear that the general level of prices depends directly upon five factors, viz., (i.) The amount of money in circulation. (ii.) Its velocity of circulation.

[^3](iii.) The amount of credit money in circulation. (iv.) Its velocity of circulation, and ( $v$.) The total quantity of goods sold, that is the volume of trade. It is obvious that if any one of these factors change in magnitude, there must result a change in the general level of prices; if more than one of them change, the net result on the level of prices will depend on whichever factor has a preponderating influence. The general principle stated by Professor Irving Fisher is that the price level increases with the increase of money (either currency or credit) and with the velocities of their corculation, and decreases with an increase in the volume of trade. Reaction to these factors is, however, not instantaneous, nor equally quick for each; hence the actual relation is very complex. It is, moreover, influenced by an element not susceptible of numerical evaluation, viz., the human element of faith or confidence in the stability of economic relations at a particular moment.
5. From 1896 to 1911, Prices Rising.-The main features of the graph of prices since 1896 are the general upward movement, accompanied by the rises in 1900 and 1907, with a considerable depression in the intervening years, and since 1907 a fall with a further rise to the highest point in 1911. The average levels of the indexnumber during the three quinquennia, 1896 to 1910 , were 769,844 and 943 respectively, and the corresponding values of the gold production were $£ 52,740,000, £ 65,620,000$ and $£ 89,110,000$ respectively. The association of these changes in the same direction is frequently cited as proving the inter-relation between the two phenomena. It has already been pointed out, however, that any relation which may exist is of by no means a simple character.

It is alleged that the enormous additions to the world's gold since 1890 would have caused an economic revolution unless they had been absorbed under very special' circumstances.*

The director of the United States Mint has published an estimate of the manner in which the output has been absorbed during the last twenty years. The world's industrial consumption of gold is stated to have been about $£ 114,000,000$ during the ten years from 1890 to 1899, and $£ 191,000,000$ during the eleven years 1900 to 1910 . These figures are exclusive of amounts used in Asia, Egypt, and South America, which for both monetary, industrial and other purposes, are computed to have absorbed during the latter period (1900 to 1910) about $£ 204,000,000$. The table given on page 87 shews that the total output during the second period ( 1900 to 1910) was $£ 825,770,000$. Therefore, subtracting the value of that used industrially and also the amount absorbed by Asia, Egypt and part of South America, the remainder available for coinage and bank reserves in Europe, the United States, Canada, Australasia, and parts of South America, would be $£ 430,770,000$. There is little doubt that this addition has had a considerable influence in raising prices both directly and indirectly by enabling a large extension of credit to take place. It appears, moreover, that this influence has tended to be more marked in the United States than in other countries, and it is, in fact, stated that the most rapidly rising prices are, in the main, those over which the United States of America exercises a preponderating influence, especially in regard to tin, copper,

[^4]and coltom. This contention is, to some extent, borne out by the graphs on page 78 and the tables on pages 77 and 81 . From these it may be seen that the increase in price-level in the United States since 1891 has been greater than in any other country except Germany.

In the report of the director of the United States Mint, it is pointed out that it is scarcely, conceivable, at any rate under the existing banking system, that the industrial development which has taken place in the United States during the period 1901 to 1910 could have occurred or been financed without the enlarged bank reserves which the gold output provided. In regard to the effect of this development on prices, it is alleged that the operations of large industrial and commercial trusts have accelerated the upward movement of price-levels, and it is asserted that times of rising prices are more favourable than times of falling prices for monopolists who wish to maintain prices at a high level. It should be pointed out, however, that even if the United States had not absorbed such large quantities of gold in recent years, the abundance of currency in other countries might readily (in accordance with the principles of the "quantity" theory) have made the prices of commodities, in the production of which Europe plays the chief part, rise faster and higher than they actually have done.

It is maintained that in many countries the rapid rise of trusts, conferences, pools and other forms of trade combination or agreement belongs to the recent epoch of rising prices and must be considered contributory to it.*

It.should be pointed out that, in the view of many economists, the increase in gold-production is not the main primary cause of the recent increase in prices. It is stated that the simplicity of that explanation is impaired by a crucial test, viz., the lower price of eredit which should follow the increased flow of gold into the bank reserves and stimulato the increased borrowing and the circulation through the banks. It is maintained that no such lowering of the price of credit has occurred, but that, on the contrary, the price of money has been higher than usual during the period of expanding outpue of gold. While it is admitted that the increased output of gold has been an essential constituent in the production of credit, it is stated that the utilisation of stocks, shares and vendible goods as a credit-basis has facilitated an enormous expansion in the demand for credit, so great that, in spite of the tendency of abundant gold to lower its price, that price has actually risen, and, in spite of the rise, the enhanced demand has been maintained. The cause of this increased demand for credit is said to be due to the great development of profitable economic enterprises upon a large business scale that has been taking place simultaneously in a number of new areas of enterprise. The impetus given to development in South America and North West Canada, the entering of Japan upon an era of eiterprise, and the general industrial expansion, taken in conjunction with the enlarged output of gold, are said to have involved a rapid and continuous demand for the application of large masses of capital. Moreover, the sinking of a large and growing proportion of the newly created wealth and labour of the world into developmental, but at present unremunerative, processes in the new and backward countries of the world, is said to be attended by a considerable sacrifico from

[^5]the standpoint of consumers, in a corresponding immediate rate of increase in output of food and materials. If this be so, when the development of these new countries and enterprises has matured, an increase in output and a fall of prices may then be expected to ensue.*
6. Conclasion.- In conclusion, it may be said that, in the present state of knowledge, it would seem impossible to determine with any certainty to what extent the gold-supply directly influences price-levels, but there is evidently ground for the prevalent opinion that the two are closely related. It would seen, however, that any direct influence which the gold output may have on prices, is at many periods less perceptible than the effects of war and militarism, industrial activity ard depression, seasonal and climatic influences, change in transport facilities, and methods of production consequent on scientific discovery and invention, the extension of the use of credit instruments, alternating crises in trade and financial speculation, capitalistic and industrial development and other contemporary movements.
-See "Canses of the Riso in Praces" by J. A. Holison. "The Contemporary Review." No. 569, October, 1912.

## APPENDIX I.

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## APPENDIX II.

Retail Prices in Metropolitan Towns, 1901 to 1912.

| TOWN. | 1901. | 1002. | 1903. | 1904. | 1905. | 1906. | 1907. | 1908. | 1909. | 1910. | 1911. | 1912 * |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BREAD, PER 2 Lb. Loaf. |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney | 2.8. | ${ }^{8 .} \begin{gathered}\text { d. } \\ \\ 3.1\end{gathered}$ | 8. ${ }^{\text {d }}$ | 8. $\begin{array}{r}\text { d } \\ 8.1\end{array}$ | 8. $\begin{array}{r}\text { a } \\ 3.0 \\ \\ \\ \hline\end{array}$ | 8. $\begin{array}{r}\text { dit } \\ 3.0\end{array}$ | 8. $\begin{array}{r}\text { d. } \\ 8.1 \\ \end{array}$ |  | S. $\begin{gathered}\text { d } \\ 3.6\end{gathered}$ | s. ${ }^{\text {d }}$. | ${ }^{8 .}{ }^{\text {d }}{ }^{\text {d }}$ |  |
| Meltbourne | 2.8 | 2.6 | 3.0 | 2.8 | 2.6 | 2.5 | 2.3 | 3.0 | 2.6 | 2.6 | 2.8 | 3. |
| Brisbane. | 3.0 | 8.1 | 3.3 | 3.0 | 2.7 | 2.7 | 8.1 | 3.4 | 8.5 | 3.3 | 3.5 | , |
| Adelaido. | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.0 | 3.1 | 3.1 | 3.6 | 3.0 | 3.0 | $3.1$ |
| Perth $\underbrace{\text { a }}$. | 3.0 | 3.8 | 4.4 | 3.6 | 3.5 | 3.5 | 3.1 | 3.5 | 3.1 | 3.4 | 3.5 |  |
| Hobart . | 2.8 | 2.9 | 3.4 | 3.0 | 2.6 | 2.9 | 2.9 | 3.3 | 3.4 | 3.3 | 3.1 |  |

Flour, PER 25 Lb . Bag.


TEA, PER LB.


COPPEE, PER EB.



RICE, PER LB.

| Sydney .. | 2.3 | 2.4 | 2.8 | 3.0 | 3.0 | 28 | 2.8 | 2.8 | 2.8 | 2.5 | 2.8 | 2.8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne** | 2.8 | 2.6 | 2.8 | 2.6 | 26 | 9.6 | 2.6 | 2,8 | 2.6 | 2.6 | 9.8 | 2.8 |
| Brisbane | 3.2 | 3.0 | 2.9 | 2.9 | 3.1 | 29 | 3.1 | 3.2 | 2.6 | 2.6 | 2.6 | 2.6 |
| Adelaide | 2.6 | 2.9 | 2.9 | 2.7 | 2.5 | 27 | 2.9 | 2.6 | 2.6 | 2.6 | 2.7 | 3.3 |
| Perth | 2.2 | 2.6 | 2.7 | 2.7 | 47 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 | 2.0 |
| Hobart | 3.0 | 3.0 | 3.0 |  |  | 3.0 | 3.0 | 2.9 | 2.8 | 2.7. | 2.9 | 8.0 |
| Shao, PER LD |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney | 2.5 | 2.6 | 3.0 | 2.7 | 2.7 | 3.7 | 4.0 | 3.7 | 2.5 | 2.5 | 2.5 | 3.0 |
| yfelbourne | 2.2 | 2.6 | 2.8 | 2.1 | 23 | 3.1 | 8.7 | 2.5 | 2.5 | 2.5 | 2.9 | 2.9 |
| Brisbane | 2.9 | 2.8 | 2.5 | 2.4 | 2.4 | 2.9 | 3.2 | 2.9 | 2.8 | 2.3 | 2.5 | 2.8 |
| Adelaide | 2.1 | 2.4 | 2.3 | 2.3 | 22 | 3.2 | 3.4 | 2.7 | 2.2 | 2.3 | 2.6 | 3.2 |
| Perth | 2.6 | 2.9 | 3.1 | 8.1 | 2.9 | 3.3 | 3.4 | 3.2 | 82 | 2.9 | 2.8 | 3.0 |
| Hobart | 3.7 | 3.3 | 2.8 | 2.6 | 2.7 | 3.2 | 3.9 | 3.3 | 2.4 | 2.5 | 2.9 | 3.2 |

[^6]
## Appendix.

Retail Prices m Metropolitan Towns, 1901 to 1912-contd.

| Town. | 1901. | 1902. | 1908. | 1904. | 1905. | 1906. | 1907. | 1908. | 1909. | 1910. | 1911. | 1912.* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| JAM (Australıan), PER LB. |  |  |  |  |  |  |  |  |  |  |  |  |
|  | d. | di. | d. | d. | d. | d. | di. | di. | d. | ${ }^{\text {d }}$ | d. | 6. |
| Sydney . | 3.6 | 36 | 4.1 | 86 | 4.1 | 4.1 | 38 | 38 | 3.6 | 8.6 | 4.0 | 4.4 |
| Melbourne | 8.8 | 4.0 | 39 | 4.0 | 3.9 | 41 | 40 | 3.9 | 3.8 | 40 | 4.0 | 4.0 |
| Brisbane . | 4.4 | 4.3 | 42 | 3.8 | 38 | 3.8 | 3.8 | 3.9 | 3.8 | 3.9 | 3.8 | 3.9 |
| Adelaide ... | 33 | 3.4 | 3.8 | 83 | + 8.3 | 3.3 | 3.3 | 3.8 | 3.8 | 3.3 | 3.3 | 3.4 |
| Perth . | 42 | 4.1 | 4.0 | 39 | 3.9 | 3.9 | 3.9 | 38 | 3.9 | 3.9 | 4.0 | 4.1 |
| Hobart | 4.1 | 4.1 | 4.1 | 4.2 | 4.1 | 4.1 | 4.1 | 4.1 | 4.1 | 4.3 | 4.3 | 4.2 |
| OATMEAI, PER LB, |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney | 22 | 25 | 25 | 2.2 | 2.2 | 2.3 | 2.6 | 2.8 | 9.6 |  | 28 | 3.0 |
| Melbourne | 2.2 | 2.6 | 21 | 1.6 | 1.7 | 2.1 | 2.2 | 2.4 | 2.1 | 9.4 | 22 | 2.7 |
| Brisbane . | 2.6 | 2.4 | 26 | 2.3 | 22 | 2.4 | 2.4 | 27 | 2.6 | 2.6 | 27 | 2.8 |
| Adelajde .. | 2.1 | 2.5 | 2.4 | 1.7 | 1.9 | 2.1 | 2.1 | 2.3 | 2.0 | 20 | 22 | 2.8 |
| Perth | 2.5 | 2.7 | 2.7 | 2.2 | 2.1 | 2.1 | 2.2 | 2.4 | 2.3 | 22 | 22 | 2.9 |
| Hobart. .. | 2.0 | 22 | 2.3 | 1.6 | 1.7 | 2.0 |  | 2.3 | 2.2 |  | 2.0 | 2.8 |
| Raisins, PER LB. |  |  |  |  |  |  |  |  |  |  |  |  |
| Syduey | 62 | 6.9 | 7.0 | 6.0 | 5.9 | 5.3 | 4.8 | 7.2 | 5.9 | 6.5 | 6.0 | 6.2 |
| Mielbourne | 7.7 | 6.4 | 6.3 | 5.5 | 6.0 | 6.3 | 6.4 | 64 | 62 | 6.5 | 6.6 | 6.3 |
| Brisbane . | 7.4 | 6.2 | 5.4 | 54 | 5.3 | 53 | 60 | 6.2 | 6.6 | 6.6 | 6.6 | 6.4 |
| Adelaide .. | 6.5 | 63 | 6.3 | 6.1 | 61 | 6.6 | 6.3 | 6.3 | 6.3 | 6.6 | 6.5 | 6.2 |
| Perth | 75 | 8.2 | 7.5 | 7.7 | 74 | 7.5 | 7.6 | 74 | 7.4 | 7.2 | 74 | 6.4 |
| Hobart .. | 73 | 6.7 | 6.7 | 6.4 | 6.2 | 63 | 7.1 | 6.3 | 6.2 | 6.2 | 6.2 | 6.3 |

Currants, pbr le.

| Sydney . . | 66 | 5.6 | 5.6 | 5.2 | 57 | 50 | 62 | 6.6 | 6.6 | 0.6 | 6.9 | 7.3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 6.6 | 5.6 | 53 | 5.3 | 54 | 5.6 | 62 | 6.7 | 0.6 | 68 | 6.8 | 7.1 |
| Brisbane | 78 | 6.5 | 5.8 | 5.7 | 56 | 5.6 | 6.5 | 6.7 | 7.0 | 7.0 | 7.0 | 7.2 |
| Adelatde | 7.1 | 6.8 | 6.1 | 5.7 | 5.7 | 6.3 | 6.4 | 6.8 | 7.2 | 7.2 | 7.2 | 6.7 |
| Perth | 6.5 | 8.0 | 5.5 | 6.0 | 6.1 | 6.2 | 6.7 | 66 | 6.6 | 6.7 | 7.0 | 70 |
| Hobart | 7.1 | 6.1 | 6.7 | 5.2 | 5.5 | 5.5 | 6.6 | 7.0 | 7.1 | 7.1 | 7.4 | 7.7 |
| Staroh. PfR Lb. |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney | 38 | 35 | 4.8 | 5.5 | 5.0 | 5.4 | 5.5 | 5.5 | 5.5 | 6.0 | 6.0 | 5.6 |
| Melbourne | 4.8 | 53 | 5.1 | 49 | 47 | 4.9 | 4.8 | 48 | 5.0 | 5.0 | 5.0 | 5.3 |
| Brisbane | 5.6 | 5.5 | 5.3 | 53 | 5.3 | 5.2 | 5.4 | 5.8 | 5.3 | 5.8 | 5.4 | 5.5 |
| Adelaide | 4.8 | 4.8 | 4.8 | 4.8 | 48 | 4.8 | 48 | 4.8 | 4.8 | 4.8 | 4.8 | 5.5 |
| Perth | 4.8 | 59 | 5.9 | 59 | 5.8 | 6.9 | 5.9 | 5.9 | 5.9 | 5.9 | 59 | 5.9 |
| Hobart | 6.0 | 60 | 6.0 | 6.0 | 6.0 | 60 | 6.0 | 6.0 | 60 | 60 | 6.0 | 6.0 |

blete, per dozen squares.

| Sydney .. | 8.9 | 8.9 | 89 | 8.9 | 8.9 | 8.9 | 8.9 | 89 | 8.9 | 8.9 | 8.9 | 89 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne* | 5.1 | 6.0 | 60 | 6.0 | 60 | 60 | 6.0 | 6.2 | 6.4 | 6.3 | 6.4 | 6.6 |
| Brisbane | 8.6 | 88 | 8.7 | 7.7 | 7.9 | 73 | 7.4 | 7.6 | 7.9 | 8.0 | 7.9 | 8.3 |
| Adelaide | 9.2 | 92 | 92 | 9.2 | 9.2 | 92 | 9.2 | 9.2 | 0.2 | 9.2 | 92 | 9.8 |
| Perth | 11.1 | 111 | 11.1 | 11.1 | 11.1 | 11.1 | 11.1 | 11.1 | 10.6 | 10.6 | 106 | 10.9 |
| Hobart | 9.2 | 9.2 | 9.2 | 9.2 | 0.2 | 0.2 | 9.2 | 8.5 | 0.2 | 92 | 92 | 9.0 |
| Candles, PER LB. |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney | 5.6 | 5.6 | 5.6 | 6.6 | 66 | 66 | 6.6 | 6.6 | 6.6 | 6.6 | 6.6 | 6.6 |
| Melbourne | 7.1 | 6.4 | 6.5 | 6.4 | 63 | 6.8 | 6.5 | 6.6 | 6.6 | 6.6 | 6.6 | 6.4 |
| Brisbane | 7.1 | 6.8 | 6.2 | 6.4 | 64 | 64 | 66 | 6.5 | 6.4 | 6.7 | 6.6 | 6.6 |
| Adelarde | 6.8 | 7.0 | 7.0 | 7.0 | 70 | 70 | 70 | 7.0. | 7.2 | 7.2 | 7.2 | 7.1 |
| Perth | 6.8 | 7.3 | 7.3 | 7.3 | 89 | 69 | 72 | 6.9 | 6.4 | 62 | 6.4 | 74 |
| Hobart | 4.6 | 4.6 | 4.6 | 4.6 | 4.6 | 46 | 4.8 | 4.3 | 5.4 | 5.4 | 5.4 | 5.9 |

SOAP, PER LB.

| Sydney | 2.2 | 2.2 | 2.6 | 2.7 | 2.7 | 2.7 | 2.7 | 27 | 2.7 | 2.7 | 2.7 | 3.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 3.0 | 2.9 | 3.0 | 3.0 | 2.9 | 29 | 3.1 | 30 | 3.0 | 3.1 | 3.1 | 3.6 |
| Brisbane | 2.6 | 27 | 2.5 | 2.8 | 25 | 2.3 | 2.6 | 2.5 | 2.5 | 2.5 | 2.6 | 2.4 |
| Adelaide | 2.4 | 2.4 | 2.5 | 2.6 | 2.5 | 2.5 | 25 | 2.5 | 2.5 | 2.8 | 2.8 | 2.6 |
| Perth | 2.7 | 2.6 | 2.7 | 2.7 | 2.7 | 2.8 | 8.0 | 8.0 | 2.8 | 2.8 | 3.0 | 2.9 |
| Hobart. | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.4 | 2.7 | 27 | 2.7 | 3.2 |

[^7]Retail Prices in Metropolitan Towns, 1901 to 1912-contd.

| Town | 1901. | 1902 | 1903. | 1904. | 1905. | 1906. | 1907. | 1908. | 1909.' | 1910. | 1011. | 1912.* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Potatoss, per 14 lbs. |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydrey $\quad$. |  |  |  |  |  |  |  |  |  |  |  | s. ${ }_{\text {d }}$ |
| Melboume |  |  |  |  |  |  | $1 \quad 1.9$ |  |  |
| Brisbane |  |  |  |  |  |  | 199 |  |  |
| Adetaide |  |  |  |  |  |  | $1 \quad 4.3$ |  |  |
| Perth |  |  |  |  |  |  | 19.8 |  |  |
| Hobart |  |  |  |  |  |  | 138 |  |  |
| ONIONS, PER LB. |  |  |  |  |  |  |  |  |  |  |  |  |
| SydneyMelbourne | 1.4 0.8 |  | 0.60.8 | 05 | 2.0 | 1.0 |  |  |  | 0.6 | 1.2 | 1.1 | 0.7 | 0.6 | 2.1 |
|  | 1.1 | 0.9 |  |  | 1.2 | 1.1 |  |  |  |  | 1.5 | 1.2 | 1.1 | 0.8 | 2.0 |
| Brisbane .. | 1.7 | 1.3 | 1.0 | 0.9 | 2.3 | 1.2 |  |  |  | 10 | 1.6 | 1.4 | 0.9 | 0.9 | 2.1 |
| Adelaide | 1.9 | 1.2 | 10 | 1.0 | 2.2 | 1.3 |  |  |  | 1.0 | 1.4 | 1.5 | 1.2 | 1.3 | 23 |
| Perth | 20 | 20 | 15 | 17 | 2.5 | 1.7 |  |  |  | 1.7 | 1.8 | 1.9 | 1.6 | 1.6 | 2.4 |
| Hobart | 1.3 | 13 | 1.4 | 1.1 | 2.1 | 11 | 1.1 | 1.6 | 15 | 1.5 | 1.5 | 2.5 |
| Kerosene, per abllon. |  |  |  |  |  |  |  |  |  |  |  |  |
| Mylney ${ }^{\text {Melbourne. }}$ | 10.1 | 10.1 | 10.1 | 10.8 | 10.1 | 10.1 | 101 | 118 | 118 | 11.8 | 11.8 | 10.8 |
|  | 10.11010.9 | 9.6 <br> $1 \quad 3.7$ | 105$1 \quad 1.4$ | 10.5 <br> 1 <br> 10 <br> 1.6 | $1 \begin{gathered}10.3 \\ 1\end{gathered}$ | 115 | 11.7$1 \quad 3.8$ | 1110 | 11 3  <br> 1 2 2 | 112120 | 11.311.9 | $1 \begin{array}{ll}1 & 0.3 \\ 1 & 0.5\end{array}$ |
| Brisbane |  |  |  |  |  |  |  |  |  |  |  |  |
| Adetaide | $\begin{array}{ll}1 & 8.9 \\ 1 & 4.5\end{array}$ | $\begin{array}{ll}1 & 8.7 \\ 1 & 01\end{array}$ | (1) $\begin{array}{ll}1 & 0.7 \\ 1 & 1.2 \\ 1 & 1.3\end{array}$ | $\begin{array}{r} 107 \\ 11.5 \\ \mathbf{1} \\ \hline 1.6 \end{array}$ | $\left\|\begin{array}{rr} 1 & 0.2 \\ & 11.1 \end{array}\right\|$ | 103 | $1 \quad 1.3$ | 10.7 | 110 | $\begin{array}{rr} 1 & 20 \\ 1 & 110 \\ 117 \end{array}$ | $\begin{array}{ll}1 & 08 \\ 1 & 03\end{array}$ | $\begin{array}{ll} 1 & 2.0 \\ 1 & 05 \\ 1 & 2.7 \end{array}$ |
| Perth | $1 \begin{array}{ll}1 & 0.2\end{array}$ | $1 \begin{array}{r}11 \\ 1 \\ 2.4 \\ \hline\end{array}$ |  |  |  | 11.5 | 10.6 | 11.6 | 11.6 |  |  |  |
| :Hobart | $1{ }^{1} 10.2$ |  |  |  | 0.7 | 11.3 | 11.8 | 123 | 11.9 | 1.9 | 18 |  |
| Milk, PER QUar |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney Melbourne | 4.0 | 46 | 4.5 | 3.8 | 40 | 4.0 | 4.8 | 5.0 | 48 | 4.5 | 4.4 |  |
|  | 4.0 | 40 | 40 | 4.0 | 40 | 4.0 | 4.2 | 4.6 | 4.1 | 4.0 | 4.2 | 5.0 4.8 |
| Brisbane | 3.9 | 3.9 | 3.94.0 | 3.94.0 | 3.9 | 3.9 | 3.9 | 3.9 | 3.9 | 49 | 4.9 | 4.8 |
| Adelajde | 40 | $\begin{aligned} & 40 \\ & 5.9 \end{aligned}$ |  |  | 4.0 | 4.0 | 44 | 5.0 | 50 | 6.1 | 5.9 | 5.9 |
| Perth | 5.94.1 |  | 6443 | 5943 | 5.9 | 5.9 | 59 | 6.4 | 6.4 | 64 | 64 | 6.9 |
| Hobart. |  | 5.9 4.1 |  |  | 4.4 | 4.4 | 40 | 4.4 | 4.6 | 4.6 | 4.8 | 5.0 |

BUTTER, PER LB.


## BaCON (Middle Cut), PER LB.

| Sydney | 9.1 | 10.8 | 10.5 | 10.2 | 9.3 | 98 |  | 10.3 |  | 11.7 |  |  |  | 10.3 |  | 10.0 |  | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 11.1 | 11.6 | 10.1 | 10.1 | 10.7 | 107 |  | 107 |  | 11.4 |  | 11.6 |  | 116 |  | 10.9 |  | 11 |
| Brisbane | 8.4 | 8.9 | 10.7 | 86 | 7.1 | 7.6 |  | 89 |  | 10.0 |  | 10.2 |  | 0.7 |  | 93 |  | 10. |
| Adelaide | 10.7 | 11.2 | 11.7 | 111 | 10.0 | 10.5 |  | 105 |  | 11.0 |  | 11.4 |  | 11.1 |  | 11.0 |  | 10. |
| Perth | 11.8 | 1.2 | 11.9 | 117 | 1.12 | $1 \quad 0.6$ | 1 | 0.1 | 1 | 0.2 | 1 | 0.4 | 1 | 0.2 | 1 | 0.2 |  | 10 |
| Hobart. | 10.1 | 10.2 | 11.1 | 94 | ' 84 | 8.6 |  | 9.9 |  | 10.8 |  | 10.7 |  | 10.4 |  | 10.0 |  | 10. |

Retall Prices in Metropolitan Towns, 1901 to 1912-contd.

| T0WN. | 1901. | 1909. | 1903. | 1904. | 1905. | 1906. | 1907. | 1908. | 1909. | 1910. | 1911. | 1912** |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bagon (Shoulder), PER LD. |  |  |  |  |  |  |  |  |  |  |  |  |
| Sydney | 8. $\begin{gathered}\text { d. } \\ 6.7\end{gathered}$ | s.d. <br> 8 | ${ }^{8 .} \begin{aligned} & \text { d. } \\ & 7.8\end{aligned}$ | ${ }^{8 .}$ d. ${ }^{\text {d }} 0$ | ${ }^{8} \begin{array}{r}\text { d. } \\ 7.0 \\ \\ 0.1\end{array}$ | s. ${ }^{\text {d }}$ 8.2 | 8. $\begin{gathered}\text { d } \\ 6.7\end{gathered}$ | ${ }^{8 .} \begin{gathered}\text { d } \\ 7.1 \\ \\ \\ \text { d }\end{gathered}$ | s. $\begin{array}{r}\text { d. } \\ 6.7\end{array}$ | ${ }^{8 .} 8.7$ | ${ }^{8 .} \begin{gathered}\text { d } \\ 6.8\end{gathered}$ | s. ${ }^{\text {d }}$. 8.8 |
| Melbourne | 6.5 | 6.5 | 7.0 | 7.0 | 6.1 | 6.2 | 6.1 | 6.5 | 6.9 | 68 | 6.1 | 6.8 |
| Brisbane | 5.6 | 5.9 | 7.3 | 5.7 | 5.2 | 58 | 6.3 | 7.0 | 6.8 | 6.8 | 6.4 | 7.1 |
| Adelalde | 6.2 | 6.3 | 6.4 | 6.3 | 5.8 | 61 | 6.1 | 6.1 | 6.7 | 5.7 | 6.1 | 6.5 |
| Perth | 8.9 | 9.4 | 9.2 | 9.1 | 9.8 | 8.4 | 7.8 | 8.2 | 8.4 | 8.3 | 8.6 | 7.8 |
| Hobart | 7.0 | 7.2 | 7.3 | 6.1 | 5.6 | 5.8 | 6.8 | 7.4 | 7.5 | 7.2 | 6.8 | 8.8 |

HaM, PER LB.

| 8ydney |  | 11.0 | 1 |  | 1 | 0.7 | 1 | 0.0 |  | 11.3 |  | 11.9 |  | 0.3 | 1 | 1.5 |  | 1.3 | 1 | 0.0 |  | 0.6 |  | 11.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne |  | 11.7 |  | 11.7 | 1 | 0.4 | 1 | 0.2 |  | 10.7 |  | 10.7 |  | 10.7 |  | 11.2 |  | 11.7 | 1 | 0.0 |  | 11.0 | 1 | 0.2 |
| Brisbane |  | 11.7 |  | 11.9 | 1 | 1.4 |  | 11.8 |  | 10.8 |  | 11.2 | 1 | 0.0 | 1 | 0.7 | 1 | 1.2 | 1 | 0.7 | 1 | 0.7 | 1 | 2.1 |
| Adelajde |  | 11.7 | 1 | 0.5 | 1 | 1.3 | 1 | 0.8 | 1 | 0.0 | 1 | 0.8 | 1 | 0.4 | 1 | 09 | 1 | 1.0 | 1 | 0.9 | 1 | 08 |  | 11.7 |
| Perth |  | 0.7 | 1 | 2.6 | 1 | 3.1 | 1 | 2.8 | 1 | 1.5 | 1 | 2.0 | 1 | 0.9 |  | 1.1 |  | 1.6 |  | 1.4 |  | 1.4 | 1 | 1.4 |
| Hobar |  | 0.6 | 1 | 0.4 | 1 | 1.4 | 1 | 0.6 | 1 | 0.2 | 1 | 02 | 1 | 0.6 | 1 | 1.6 | 1 | 0.8 | 1 | 0.8 | 1 |  | 1 | 0.8 |

Beef, Fresh, Sireoln, per lb.


Beef, Frissh, Rib, per lb.

| Sydney .- | 4.7 | 6.1 | 5.0 | 4.5 | 4.5 | 4.5 | 4.6 | 4.6 | 4.5 | 4.5 | 4.5 | 4.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 5.2 | 5.8 | 5.1 | 4.7 | 46 | 4.6 | 4.9 | 5.0 | 4.8 | 4.6 | 4.2 | 5.0 |
| Brisbane . . | 4.1 | 4.3 | 4.1 | 3.9 | 3.7 | 4.1 | 3.9 | 4.1 | 3.4 | 3.7 | 3.7 | 3.3 |
| Adelaide | 5.4 | 5.0 | 5.0 | 4.8 | 4.9 | 49 | 5.0 | 5.0 | 4.9 | 4.7 | 4.7 | 4.8 |
| Perth | 5.5 | 5.8 | 6.1 | 5.4 | 5.7 | 6.1 | 5.7 | 5.6 | 5.7 | 0.0 | 6.3 | 6.6 |
| Hobart. | 5.6 | 6.4 | 63 | 5.8 | 6.0 | 60 | 5.7 | 5.9 | 60 | 59 | 5.6 | 5.4 |

Beef, Feesh, Flank, per fb.

| Sydney | 3.6 | 4.7 | 3.9 | 3.4 | 3.4 | 3.4 | 3.5 | 3.5 | 3.4 | 3.4 | 3.4 | 3.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melboume | 4.5 | 5.2 | 4.5 | 4.0 | 40 | 3.9 | 42 | 4.4 | 4.2 | 3.9 | 3.6 | 4.0 |
| Brisbane | 5.5 | 5.5 | 5.5 | 5.5 | 5.5 | 5.5 | 5.5 | 5.5 | 46 | 3.7 | 4.6 | 3.4 |
| Adelatde | 4.1 | 4.1 | 4.0 | 4.0 | 4.0 | 4.0 | 4.1 | 4.1 | 3.9 | 3.8 | 3.8 | 4.2 |
| Pertl) | 5.9 | 6.9 | 7.2 | 6.3 | 6.8 | 7.0 | 6.0 | 65 | 6.6 | 69 | 7.3 | 6.1 |
| Hobart | 4.0 | 46 | 4.5 | 4.2 | 42 | 4.0 | 4.1 | 41 | 42 | 3.9 | 3.6 | 3.9 |

BEDF, FRESH, Shin, PER LD. -

| Sydney . . | 3.3 | 4.3 | 3.5 | 3.0 | 3.0 | 3.0 | 32 | 3.2 | 8.0 | 3.0 | 3.0 | 3.4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Afelbourne | 3.9 | 4.4 | 3.8 | 8.6 | 3.5 | 3.4 | 3.8 | 3.9 | 3.7 | 3.4 | 3.1 | 3.6 |
| Brisbane | 3.5 | 3.3 | 3.1 | 31 | 3.1 | 3.1 | 3.0 | 2.9 | 2.4 | 27 | 2.7 | 3.0 |
| Adelaide | 3.9 | 3.6 | 3.5 | 3.4 | 3.6 | 8.5 | 35 | 3.5 | 3.9 | 3.4 | 3.4 | 3.7 |
| Perth | 4.9 | 5.7 | 5.7 | 4.9 | 4.9 | 49 | 4.9 | 4.9 | 4.9 | 4.9 | 4.9 | 5.9 |
| Hobart | 4.0 | 4.3 | 45 | 4.1 | 4.6 | 43 | 4.1 | 4.1 | 43 | 4.0 | 4.0 | 4.5- |

STEAK, ROMLP, PER LB.

| Sydney | 7.3 | 8.7 | 8.1 | 6.7 | 6.7 | 6.7 | 7.0 | 7.0 | 6.7 | 6.7 | 6.7 | 8.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 8.0 | 8.8 | 8.4 | 8.2 | 8.2 | 8.0 | 8.4 | 8.3 | 8.0 | 7.5 | 6.9 | 8.7 |
| Brisbane | 5.3 | 6.1 | 6.3 | 5.7 | 5.6 | 6.5 | 6.3 | 8.6 | 6.2 | 6.3 | 6.1 | 6.4 |
| Adelaıde | 7.8 | 7.6 | 8.0 | 8.0 | 8.0 | 8.0 | 8.0 | 8.0 | 8.3 | 8.0 | 8.0 | 8.1 |
| Perth | 9.7 | 10.7 | 11.2 | 10.2 | 10.2 | 10.4 | 10.4 | 10.4 | 10.7 | 10.4 | 10.5 | 11.8 |
| Hobart | 8.0 | 8.6 | 8.6 | 8.1. | 8.4 | 8.3 | 8.4 | 8.0 | 8.4 | 8.1 | 8.1 | 8.4 |

Steak, Shoulder, per Lb.

| Syduey | 4.1 | 5.0 | 4.3 | 3.4 | 8.4 | 9.4 | 3.6 | 3.6 | 3.4 | 3.4 | 3.4 | 4.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 4.6 | 5.2 | 4.5 | 4.3 | 4.2 | 4.4 | 4.5 | 4.6 | 4.2 | 4.4 | 3.4 | 4.4 |
| Brisbane | 3.7 | 4.2 | 4.3 | 3.7 | 3.6 | 4.1 | 4.2 | 4.5 | 3.8 | 3.7 | 3.5 | 3.8 |
| Adelade | 5.4 | 5.0 | 5.0 | 4.9 | 4.9 | 5.0 | 4.5 | 4.4 | 4.5 | 4.1 | 4.4 | 4.5 |
| Perth | 5.6 | 6.8 | 7.2 | 6.3 | 7.1 | 7.5 | 7.0 | 6.5 | 6.9 | 7.3 | 7.3 | 7.2 |
| Hobart. | 5.1 | 5.6 | 5.6 | 5.4 | 5.3 | 5.1 | 5.8 | 5.2 | 5.4 | 5.2 | 5.1 | 5.3 |

- First 9 months of 1912.

Retail Prices in Metropolitan Towns, 1901 to 1912-contd..


PORE, LOIN, PER LB.

| Sydmey . | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.0 | 6.8 | 7.3 | 75 | 8.0 | 80 | 8.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mellourne | 6.4 | 7.4 | 7.0 | 6.6 | 8.0 | 6.0 | 6.4 | 6.6 | 7.0 | 8.5 | 8.0 | 7.1 |
| Brisbane | 6.3 | 6.9 | 7.5 | 70 | 6.4 | 6.6 | 6.4 | 6.6 | 6.8 | 68 | 7.3 | 7.2 |
| Adelatde | 7.7 | 7.7 | 7.3 | 72 | 7.3 | 6.9 | 6.7 | 7.2 | 7.2 | 7.2 | 7.2 | 7.9 |
| Perth | 6.9 | 7.2 | 6.9 | 7.2 | 7.4 | 7.4 | 7.9 | 78 | 7.7 | 7.2 | 7.2 | 9.0 |
| Hobart | 6.6 | 6.6 . | 6.9 | 68 | 66 | 6.6 | 6.6 | 6.9 | 6.6 | 0.6 | 6.7 | 6.5 |

Pork, Belly, pER EB.

| Sydney | 5.5 | 59 | 5.9 | 5.9 | 5.9 | 5.9 | 65 | 7.1 | 7.3 | 7.5 | 7.5 | 7.6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Melbourne | 6.2 | 7.3 | 7.0 | 6.6 | 8.1 | 6.0 | 6.4 | 6.6 | 70 | 6.5 | 6.1 | 6.9 |
| Brisbane | 5.0 | 0.5 | 6.0 | 5.7 | 5.5 | 5.1 | 5.2 | 5.4 | 5.7 | 5.5 | 6.5 | 6.0 |
| Adelande | 7.6 | 7.6 | 7.2 | 7.1 | 7.2 | 6.9 | 6.6 | 7.1 | 7.1 | 7.1 | 7.1 | 7.8 |
| Perth | 7.0 | 7.8 | 7.0 | 7.3 | 7.5 | 7.5 | 7.9 | 7.8 | 7.6 | 7.1 | 7.1 | 8.1 |
| Hobart . | 6.7 | 7.0 | 6.7 | 6.9 | 66 | 8.9 | 6.9 | 6.7 | 6.5 | 6.6 | 67 | 6.5 |

PORK, CROPS, PER LB.

| Sydney | 7.0 | 6.7 | 6.7 | 67 | 6.7 | 6.7 | 7.4 | 7.9 | 8.1 | 8.6 | 8.6 | 8.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mielbourne | 7.0 | 8.1 | 7.6 | 7.2 | 6.6 | 6.4 | 7.0 | 7.2 | 7.6 | 7.1 | 6.6 | 7.8 |
| Briguane | 6.7 | 7.3 | 8.1 | 7.0 | 6.7 | 6.8 | 6.6 | 6.8 | 7.0 | 7.2 | 7.7 | 7.9 |
| Adelaide | 80 | 80 | 7.8 | 7.8 | 7.8 | 7.3 | 6.8 | 7.3 | 7.8 | 7.3 | 7.3 | 8.0 |
| Perth | 86 | 8.7 | 8.5 | 8.7 | 9.0 | 9.0 | $\cdot 9.1$ | 9.4 | 9.3 | 8.7 | 8.7 | 0.6 |
| Hobart | 7.4 | 7.5 | 7.4 | 6.8 | 6.7 | 6.8 | 7.1 | 6.9 | 7.2 | 6.8 | 68 | 7.0 |

* First 9 months of 1912.


## APPENDIX III.

## Carrent Retail Pricés in Metropolitan and Conntry Towns, 1912.*

| TOWN. | Bread <br> 2 lbs. | Flour | Tea | Cotfee per lb | Sugar | Rice per lb | Sago per lb, | Jam per lb. | Oatmeal <br> per lb. | $\left.\begin{gathered} \mathbf{R}_{\text {fus. }} \\ \text { ins } \\ \text { per lb. } \end{gathered} \right\rvert\,$ | Cur <br> rants <br> per lb. | Starch |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 16. ${ }^{\text {d }}$, | s. d. |  | d. | d. | d. | $d$. | d. | d. | d. | $d$. |
| Sydney | 3.3 | 210.4 | $1{ }^{1} 4.0$ | $1{ }^{+1} 6.8$ | 2.8 | 2.8 | 3.0 | 4.4 | 3.0 | B, 2 | 7.3 | 5.6 |
| Newcastle |  | 211.0 | 135.4 | 15.6 | 2.9 | 32 | 3.2 | 4.4 | 2.9 | 7.0 | 7.5 | 5.6 |
| Broken Hill | 3.5 | $2 \begin{array}{ll}2 & 11.7\end{array}$ | 180.2 | 17.8 | 3.3 | 4.0 | 4.1 | 4.5 | 8.5 | 7.1 | 7.4 | 6.9 |
| Goulburn . . | 3.2 | 29.9 | $1 \quad 5.9$ | 16.6 | - 3.0 | 3.1 | 3.7 | 4.6 | 31 | 7.6 | 7.7 | 6.0 |
| Bathurst . . |  | 26.7 | $1 \begin{array}{ll}1 & 5.8\end{array}$ | 16.0 | 3.1 | 3.0 | 34 | 4.9 | 3.0 | 7.2 | 7.1 | 6.1 |
| Melbourne | 9.0 | $2 \quad 6.7$ | t 2.7 | 16.6 | 3.0 | 2.8 | 2.9 | 4.0 | 2.7 | 6.3 | 7.1 | 5.3 |
| Ballarat | 3.1 | 264 | 13.0 | 15.9 | . 3.0 | 29 | 2.9 | - 3.6 | 2.7 | 5.8 | 6.7 | 5.4 |
| Bendigo | 3.2 | 260 | $1 \begin{array}{ll}1 & 2.2\end{array}$ | 1 6.t | 3.2 | 29 | 3.0 | 3.5 | 28 | 5.8 | 7.0 | 5.3 |
| Geelong | 3.2 | 294 | 12.6 | 15.2 | 3.0 | 2.6 | 3.0 | 8.7 | 2.9 | 6.2 | 7.5 | 5.1 |
| Warcnambool | 33 | 28.3 | 1.3 .2 | 18.1 | 29 | 2.7 | 3.0 | 38 | 2.8 | 6.3 | 6.9 | 5.8 |
| Brisbane | 3.5 | 31.7 | 14.3 | 17.1 | 8.0 | 2.3 | 2.8 | 3.9 | 2.8 | 6.4 | 7.2 | 5.5 |
| Toownomba | 3.5 | 38 | $1 \quad 3.0$ | 159 | 31 | 30 | 3.2 | 4.8 | 3.0 | 7.1 | 7.7 | 6.0 |
| Rockhampt'n | 38 | 30.9 | 160 | 166 | 2.8 | 2.8 | 8.0 | 44 | 2.9 | 0.2 | 7.0 | 6.0 |
| Charters Towers | 4.5 | 38 | $1 \begin{array}{ll}1 & 7.6\end{array}$ | 17.8 | 3.5 | 3.7 | 4.0 | 4.8 | 3.3 | 6.8 | 7.7 | 6.0 |
| Warwick . | 3.7 | $3 \quad 39$ | $1 \quad 6.1$ | 16.0 | 3.1 | 3.0 | 3.4 | 4.8 | 31 | 8.2 | 7.6 | 6.0 |
| Adelajue .. | 3.5 | 29.1 | 14.4 | 15.9 | 2.9 | 3.3 | 3.2 | 3.4 | 2.8 | 6.2 | 6.7 | 5.5 |
| Kallina |  |  |  |  |  |  |  |  |  |  |  |  |
| Moontra \& Wallaroo | 3.3 | $2 \quad 6.1$ | 134 | 16.2 | 2.9 | 8.1 | 3.1 | 8.5 | 2.8 | 6.6 | 6.6 | 5.9 |
| Port Pirie | 3.0 | 291 | $1 \quad 6.0$ | $1 \quad 7.9$ | 3.8 | 3.4 | 3.7 | 3.9 | 8.1 | 7.1 | 7.5 | 6.2 |
| Mt. Gambier | 3.0 | $2 \begin{array}{lr}2 & 9.9\end{array}$ | $1 \begin{array}{ll}1 & 4.2\end{array}$ | $1 \quad 7.7$ | 3.0 | 3.0 | 3.2 | 4.0 | 2.9 | 6.3 | 7.1 | 5.7 |
| Petersburg | 8.4 | 211.2 | 1.5 .6 | 17.5 | 3.2 | 3.5 | 3.0 | 4.2 | 8.0 | 7.4 | 6.9 | 6.2 |
| Perth and |  |  |  |  |  |  |  |  |  |  |  |  |
| Fremantle | 35 | 284 | 113.8 | 17.2 | 3.0 | 2.8 | 8.0 | '41 | 2.0 | 6.4 | 7.0 | 5.8 |
| Kalgoorlie s Boulder | 5.0 | $3 \begin{array}{ll}3 & 3.1\end{array}$ | II 7.3 | 1 9.4 | 3.8 | 39 | 4.0 | 5.2 | 9.4 | 9.3 | 8.1 | 7.0 |
| Mid. Junction |  |  |  |  |  |  |  | 4.0 |  |  |  |  |
| \& Guildford | 3.5 | $\begin{array}{rr}2 & 9.0 \\ 9 & 10.6\end{array}$ | $1 \begin{array}{ll}1 & 3.8 \\ 1 & 3.7\end{array}$ |  | 3.0 | 2.9 | 30 | 4.0 | 3.9 | 6.0 | 7.3 7.3 | 6.0 |
| Bunbury .. Geraliton | 3.5 4.6 | $\begin{array}{ll}2 & 10.6 \\ 2 & 11.1\end{array}$ | $\begin{array}{ll}1 & 3.8 \\ 1 & \mathbf{5 . 4} \\ & \end{array}$ | 1 6.0 <br> 1 3.4 | 3.2 33 | 3.1 3.0 | 4.10 | 4.2 4.3 | 3.9 | 6.7 7.8 | 7.3 7.9 | 6.1 6.0 |
| Hobart | 3.5 | 30.5 | 13.4 | 1-6.0 | 3.0 | 3.0 | 3.2 | 4.2 | 2.8 | 6.3 | 7.7 | 6.0 |
| Laurceston | 3.3 | 2rrall | 12.5 | L-5.2 | 2.7 | 2.9 | 2.8 | 3.9 | 2.5 | 6.1 | 7.1 | 5.4 |
| Zeehan | 3.7 | 211.3 | $1 \begin{array}{ll}1 & 4.2\end{array}$ | 1 | 3.0 | 3.0 | 3.2 | 4.2 | 2.9 | 6.6 | 7.1 | 5.5 |
| Beaconsfield | 8.2 | 210.6 | 12.8 | $1 \quad 5.8$ | 3.0 | 3.0 | 3.4 | 4.3 | 2.8 | 7.5 | 7.1 | 5.9 |
| Queenstown | 3.7 | 3 3 02 | 156 | 14.7 | 2.9 | 3.0 | 3.1 | 4.0 | 2.7 | 7.6 | 7.8 | 5.7 |
| Weighted Average | 3.3 | $12 \quad 9.4$ | 113.8 | 18.4 | 3.0 | 9.9 | 3.0 | 4.1 | 2.8 | 6.4 | 7.2 | 5.6 |

[^8]Current Retail Prices in' Metropolitan and Country Towne, 1912,*-contd.

| Town. | Blue. $\mathrm{dz} \mathrm{c}_{\mathrm{s}}^{\mathrm{sq}},$ | Candle | Soap per lb. | Pota toes. 14 lvs. | Onions per lb. | Kerosene gallon | Milk <br> quart | Butter | Cheese $\text { per } \mathrm{ib} \text {. }$ |  | Bacon Middle per tb. | Bacon Shou* der. per lb. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 8. d. | d. | d. |  | d. | c. d. | $d$. | b. d. | s. 'd. | s. d. |  | d. |
| Sydney | 8.9 | 6.6 | 3.0 | $1 \quad 6.9$ | 2.1 | 10.8 | 5.0 | 13.6 | 11.7 | 18.1 | 10.5 | 6.8 |
| Newcastle | 10.0 | 7.0 | 3.6 | 17.5 | 2.5 | 12.1 | 4.3 | $1 \begin{array}{ll}1 & 8.7 \\ 1\end{array}$ | 11.7 | 8.3 | 10.8 | 9.5 |
| Broken Hill | 10.7 | 8.5 | 3.2 | 18.8 | 2.8 | 19.6 | 6.2 | 18.8 | 10.5 | 16.4 | 11.3 | 9.1 |
| Goutburn . | 11.2 | 6.4 | 3.2 | 13.7 | 2.6 | 13.2 | 49 | 13.9 | 11.6 | 15.9 | 10.6 | 7.8 |
| Bathurst | 11.2 | 7.3 | 29 | 12.4 | 2.8 | $1 \quad 6.3$ | 4.4 | $1 \begin{array}{ll}1 & 3.7\end{array}$ | 11.7 | 15.9 | 11.4 | 9.0 |
| Melbourne | 6.6 | 6.4 | 3.6 | 11.9 | 20 | 10.3 | 4.8 | $1 \begin{array}{ll}1 & 4.2\end{array}$ | 11.1 | $1{ }^{1} 6.6$ | 11.8 | 6.8 |
| Ballarat | 6.8 | 5.9 | 2.9 | 11.9 | 20 | 1.1 .5 | 3.9 | 11 3.1 | 10.9 | $1 \quad 2.9$ | 11.5 | 6.9 |
| Bendigo | 6.4 | 6.6 | 3.1 | 13.3 | 2.1 | 12.5 | 4.8 | 136 | 11.3 | $1 \begin{array}{ll}1 & 3.7\end{array}$ | 102 | 6.2 |
| Geelong | 6.3 | 6.0 | 3.0 | 116 | 1.9 | 12.6 | 4.5 | $1 \begin{array}{ll}1 & 3.4\end{array}$ | 10.7 | $1 \begin{array}{ll}1 & 4\end{array}$ | 11.3 | 6.9 |
| Warcnambool | 7.1 | 6.2 | 2.9 | 13 | 1.7 | 13 | 4.0 | 14.2 | 10.6 | 13.2 | 10.4 | 6.8 |
| Brisbane | 8.3 | 6.6 | 2.4 | 19.9 | 2.1 | 10.5 | 4.8 | $1 \begin{array}{ll}1 & 3.0\end{array}$ | 11.2 | 186.9 | 10.3 | 7.1 |
| Toowoomba | 10.0 | 7.2 | 3.0 | 110.6 | 2.4 | + +4.9 | 4.2 | $1 \begin{array}{ll}1 & 3.4 \\ 1\end{array}$ | 110 | 15.7 | 9.9 | 7.5 |
| Rockhampt'n | 9.5 | 6.9 | 25 | 19.8 | 23 | 11.5 | 49 | $1 \quad 2.5$ | 11.8 | $1 \begin{array}{ll}1 & 7.3\end{array}$ | 0.3 | 7.6 |
| Towers | 10.3 | 7.8 | 2.8 | 24.3 | 2.9 | 16.3 | 48 | 155 | $1 \quad 0.7$ | 10.3 | 115 | 8.8 |
| Warwick . | 11.6 | 7.3 | 2.9 | 19.6 | 27 | 16.0 | 42 | 13.6 | 10.9 | 14.6 | 10.0 | 8.4 |
| Adelaide .- | 9.3 | 7.1 | 2.6 | 14.3 | 2.3 | 12.0 | 5.9 | 15.8 | 11.5 | 13.9 | 10.9 | 6.5 |
| Kadina |  |  |  |  |  |  |  |  |  |  |  |  |
| Wallaroo .. | 9.6 | 7.3 | 3.1 | 14.8 | 2.5 | 1.3 .2 | 00 | 15.2 | 11.6 | 120 | 10.7 | 0.2 |
| Port Pirje | 11.6 | 8.5 | 2.5 | $1 \begin{array}{ll}1 & 5.8\end{array}$ | 2.6 | $1 \begin{array}{ll}1 & 4.1\end{array}$ | 58 | $1 \begin{array}{ll}1 & 5.9\end{array}$ | 108 | $1 \begin{array}{ll}1 & 2.8 \\ 1 & 0.8\end{array}$ | 11.7 | 93 |
| Mit. Gambier | 11.4 | 7.8 | 30 | $1 \begin{array}{ll}1 & 1.9\end{array}$ | 22 | 138 | 3.4 | 12.5 | 10.1 | $1 \begin{array}{ll}1 & 0.1\end{array}$ | 10.6 | 8.1 |
| Petersburg | 11.7 | 7.9 | 3.5 | 15.4 | 2.8 | 150 | 44 | $1 \begin{array}{ll}1 & 4.4\end{array}$ | 11.6 | 1.2 .0 | 1.1.5 | 9.8 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Perth and Fremantle | 10.9 | 7.4 | 2.0 | 19.8 | 2.4 | $1 \cdot 0.5$ | 0.0 | 14.9 | 11.9 | 110.4 | 10.3 | 7.8 |
| Kalgoorlie \& |  |  |  |  |  |  |  | 1.4 .8 | 12.0 | 110.4 | $1-0.3$ | 7.8 |
| Boulder | 10.0 | 0.5 |  | $2 \quad 3.6$ | 33 | $1 \quad 9.7$ | 9.0 | 1.7 .3 | $1 \quad 1.5$ | $2 \quad 2.4$ | $1 \quad-1.7$ | 9.6 |
| Mid.Junction 8. GutidFord |  |  |  |  |  |  |  |  |  |  |  |  |
| \& Gutidford | 10.8 | 8.2 | 3.1 2.7 | $\begin{array}{rr}2 & 0.0 \\ 1 & 20.7\end{array}$ | 2.7 | 1 0.5 <br> 1 10 <br> 1  | 5.0 | IV 5.5 | $1 \begin{array}{ll}1 & 0.1 \\ 1 & 0.1\end{array}$ | $\left\lvert\, \begin{array}{rr} 1 & 10.1 \\ 1 & 8.9 \end{array}\right.$ | $\begin{array}{ll}1 & 0.3 \\ 1 & 0.0\end{array}$ | 8.7 8.4 |
| Bunbury . . | 11.2 | 8.5 | 2.7 | $\begin{array}{ll}1 & 10.7 \\ 2 & 1.0\end{array}$ | 2.8 | $1 \begin{aligned} & 1 \\ & 1\end{aligned} 10$ | 5.6 | 15 | $1 \begin{array}{ll}1 & 0.1 \\ 1 & 0.9\end{array}$ | $\begin{array}{rr} 1 & 8.9 \\ \hline 17 \end{array}$ | $1 \begin{array}{ll}1 & 0.0 \\ 1 & 1.3\end{array}$ | 8.4 |
| Geraldton | 10.0 | 8.7 | 2.7 | 21.0 | 2.4 | 1123 | 60 | 17.0 | 10.9 | 111.7 | 11.3 | 9.6 |
| Hobart | 9.0 | 5.9 | 39 | $1 \begin{array}{ll}1 & 3.8\end{array}$ | 2.5 | $1 \begin{array}{ll}1 & 8.7\end{array}$ | 5.0 | 14.2 | 11.5 | 1.5 .6 | 10.7 | 6.8 |
| Tarnnceston | 7.3 | 5.7 | 28 | 137 | 2.2 | $1 \begin{array}{ll}1 & 3.4 \\ 1 & 3 .\end{array}$ | 4.6 | 183 | 10.5 | $1 \cdot 4.0$ | 10.0 | 7.7 |
| Zeeltan | 9.1 | 7.0 | 3.1 | 17.3 | 23 | $1 \begin{array}{ll}1 & 3.2\end{array}$ | 5.6 | $1 \begin{array}{ll}1 & 6.0\end{array}$ | 11.5 | 17.2 | 10.0 | - 7.8 |
| Beaconsfield | 9.4 | 7.2 | 4.2 | 14.0 | 2.5 | $1 \begin{array}{ll}1 & 3.4\end{array}$ | 4.8 | 178 | 10.9 | $1 \begin{array}{ll}1 & 4.8\end{array}$ | 9.7 | 8.2 |
| Queenstown | 8.7 | 7.7 | 3.3 | 16.4 | 2.1 | 15.4 | 5.6 | 14.3 | 10.6 | 17.2 | 10.0 | 8.2 |
| Weighted Average | 8.4 | 6.7 | 3.1 | 115.4 | 2.2 | $1 \begin{array}{ll}1 & 1.4\end{array}$ | 8.1 | 14.2 | 11.5 | 10.8 | 11.1 | 7.1 |

[^9]Current Retail Prices in Metropolitan and Country Towns, 1912.*-contd,

| TOWN. | Ham <br> per $\mathrm{Ib}^{2}$. | Beef Fresh Sirloin <br> per ib. | $\substack{\text { Beef } \\ \text { Frefh } \\ \text { Rfb } \\ \text { per lb. }}$ | Beef Fresh Flank per lb. | Beef Fregh Shin <br> per lb. | Steak Rump <br> per lb | $\begin{aligned} & \text { Steak } \\ & \text { per lb'lder } \end{aligned}$ | Steak Buttock per lb. | Beet Co'n'd round <br> per Jb. | Beef <br> Co'n'd brigket with bone per lb. | Beef $C_{0} \mathbf{n}^{*} \mathrm{~d}$ brisket without bone <br> per lb | Matt'a <br> Leg <br> per lb. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | d. | d | $a$ | d | d. | d. | d. | d | d. | d | d |
| Sydney | 11.5 | 5.9 | 4.7 | 3.9 | 3.4 | 8.2 | 4.1 | 4.2 | 4.5 | 3.0 | 3.9 | 4.0 |
| Newcastle | 11.9 | 5.7 | 5.0 | 3.9 | 3.8 | 7.3 | 4.3 | 4.4 | 4.7 | 3.6 | 5.0 | 4.9 |
| Broken Hill | 10.6 | 6.6 | 5.6 | 3.9 | 5.2 | 10.0 | 6.2 | 6.6 | 6.6 | 4.2 | 5.6 | 6.1 |
| Goulburn . . | 11.5 | 5.2 | 4.2 | 8.8 | 3.5 | 6.1 | 4.2 | 4.2 | 5.2 | 39 | 4.8 | 4.2 |
| Bathurst . | 10.6 | 4.4 | 3.9 | 3.2 | 3.3 | 5.9 | 3.8 | 4.0 | 4.1 | 3.3 | 3.8 | 4.1 |
| Melbourne | 10.2 | 6.2 | 5.0 | 4.0 | 3.6 | 8.7 | 4.4 | 50 | 5.1 | 3.1 | 4.1 | 4.2 |
| Ballarat | 10.2 | 7.0 | 6.1 | 4.9 | 4.4 | 9.7 | 5.6 | 5.9 | 6.9 | 4.2 | 5.9 | 4.9 |
| Bendigo | 11.3 | 5.6 | 5.2 | 3.6 | 4.2 | 8.0 | 5.0 | 5.3 | 5.3 | 3.5 | 4.7 | 4.8 |
| Geelong ... | 11.8 | 5.8 | 5.4 | 4.0 | 3.8 | 7.8 | 4.3 | 5.2 | 5.5 | 3.2 | 4.4 | 4.4 |
| Wartnambool | 11.1 | 5.5 | 4.8 | 3.3 | 4.1 | 6.9 | 4.0 | 48 | 4.9 | 3.1 | 4.0 | 4,0 |
| Brisbane . . | 12.1 | 4.4 | 3.3 | 3.4 | 8.0 | 6.4 | 3.8 | 3.7 | 4.4 | 2.9 | 8.9 | 4.6 |
| Toowoomba | 124 | 4.9 | 3.1 | 2.1 | 2.5 | 5.8 | 29 | 2.9 | 4.4 | 2.5 | 3.9 | 4.5 |
| Rockhampt's | 12.9 | 5.7 | 4.8 | 3.4 | 2.8 | 5.9 | 40 | 3.7 | 50 | 8.9 | 4.6 | 5.3 |
| Charters Towers | 13.8 | 5.0 | 3.5 | 3.6 | 4.0 | 6.0 | 4.0 | 4.0 | 4.5 | 3.5 | 4.5 | 5.5 |
| Warwick . | 12.5 | $\mathbf{5 . 0}$ | 4.0 | 4.0 | 4.0 | 5.0 | 4.0 | 4.0 | 5.0 | 4.0 | 4.4 | 5.0 |
| Adelicide Kadina Mroonta | 11.7 | 5.8 | 4.8 | 4.2 | 3.7 | 8.1 | 4.5 | 4.9 | 5.7 | 3.6 | 4.5 | 4.4 |
| Wallaroo .. | 11.4 | 5.6 | 5.4 | 3.9 | 4.5 | 7.0 | 5.4 | 5.4 | 5.6 | 4.2 | 5.0 | 4.0 |
| Port Pirie | $1 \quad 0.7$ | 6.9 | 4.9 | 3.1 | 4.4 | 7.9 | 5.4 | 5.6 | 8.0 | 4.0 | 4.9 | 4.9 |
| Mt. Gambier | 11.8 | 5.2 | 4.6 | 3.6 | 4.0 | 5.9 | 4.0 | 4.0 | 5.2 | 4.0 | 5.0 | 4.1 |
| Petersburg | 1 0.9 | 5.3 | 5.0 | 4.8 | 4.7 | 7.1 | 5.0 | 5.0 | 5.5 | 4.1 | 4.9 | 4.9 |
| Perth and Fremantle | $1 \quad 1.4$ | 7.7 | 6.6 | 6.1 | 5.9 | 11.8 | 72 | 7.2 | 7.0 | 4.6 | 6.0 | 7.8 |
| Kalgoorlie Boutder | $1 \begin{array}{ll}4.6\end{array}$ | 8.9 | 7.8 | 6.8 | 8.7 | 11.9 | 8.6 | 8.7 | 8.7 | 6.3 | 8.1 | 8. |
| Mid. Junction |  | 8.8 |  |  |  | 11.9 | 8.6 | 8.7 | 8.7 | 6.3 | 8.1 |  |
| © Guildiord | $1 \begin{array}{ll}1 & 1.4\end{array}$ | 8.3 | 7.5 | 4.9 | 7.0 | 10.5 | 8.0 | 7.8 | 7.6 | 5.4 | 6.6 | 8.3 |
| Bunbury .. | 11.0 | 9.0 | 8.0 | 5.7 | 7.2 | 10.0 | 8.1 | 8.1 | 8.3 | 6.1 | 7.3 | 9. |
| Geralditon | 12.5 | 7.6 | 6.6 | 4.9 | 6.3 | 9.1 | 6.9 | 7.2 | 7.2 | 5.7 | 6.4 | 7.6 |
| Hobart . . | 10.8 | 6.4 | 5.4 | 3.9 | 4.5 | 8.4 | 5.8 | 6.0 | 5.9 | 3.5 | 4.4 | 5.9 |
| Launceston | 11.8 | 6.1 | 5.5 | 3.8 | 4.8 | 7.0 | 5.2 | 5.7 | 5.7 | 3.8 | 5.2 | 5.2 |
| Zeenan | 11.4 | 6.8 | 6.6 | 5.4 | 5.9 | 8.0 | 6.4 | 6.8 | 6.8 | 5.8 | 6.5 | 6.9 |
| Beaconsfield | 10.0 | 6.2 | 5.6 | 4.4 | 5.5 | 6.9 | 6.1 | 6.1 | 5.4 | 4.3 | 5.8 | 5.4 |
| Queenstown | 11.9 | 7.0 | 6.5 | 4.5 | 5.7 | 8.2 | 6.5 | 6.9 | 6.8 | 4.7 | 5.7 | 6.6 |
| Weighted Average | 10.1 | 6.0 | 49 | 4.1 | 3.8 | 8.4 | 4.6 | 4.9 | 5.2 | 3.4 | 4.4 | 4.6 |

* Average prices for first 9 months only.

Current Retail Prices in Metropolitan and Country Towns, 1912.*-contd.

| TOWN. | Mutt'n sh'Jdet per lb. | Mutt'n Lom per lb | Mutt'r Neck per lb. | Chops Loin per lb. | Chops Leg per db. | Chops Neck <br> per lb. | Pork Ieg pér lb. | Pork Loin per lb. | Pork Belly per Ib. | Pork Chops per Ib. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | d. | $d$. | $d$. | d. | $d$ | $d$ | d. | d. | d. | d. |
| Sydney | 3.4 | 4.5 | 3.6 | 5.4 | 4.7 | 4.1 | 7.8 | 8.2 | 7.6 | 8.7 |
| Newcastle | 4.1 | 4.7 | 4.0 | 4.9 | 5.1 | 4.3 | 6.4 | 7.7 | 6.8 | 7.8 |
| Broken Hill | 5.1 | 5.5 | 4.4 | 6.3 | 6.3 | 5.8 | 9.1 | 91 | 8.3 | 0.9 |
| Goulburn .- | 3.6 | 4.2 | 2.9 | 4.2 | 4.2 | 3.0 | 5.8 | 6.0 | 5.6 | 6.0 |
| Bathurat . ${ }^{\text {- }}$ | 3.5 | 4.0 | 28 | 41 | 4.2 | 3.6 | 5.8 | 61 | 5.9 | 6.2 |
| Melbourne | 3.4 | 42 | 80 | 5.2 | 4.7 | 3.5 | 6.4 | 7.1 | 0.9 | 7.3 |
| Ballarat | 4.0 | 51 | 3.5 | 5.5 | 5.9 | 4.3 | 7.3 | 81 | 8.3 | 8.2 |
| Bendigo | 3.5 | 4.7 | 33 | 5.3 | 5.3 | 4.2 | 6.3 | 6.6 | 60 | 7.0 |
| Geelong | 3.6 | 45 | 3.5 | 48 | 4.8 | 3.8 | 6.6 | 7.2 | 7.1 | 7.4 |
| Warrnambool | 4.0 | 4.9 | 3.6 | 5.1 | 5.0 | 4.0 | 6.0 | 63 | 61 | 0.5 |
| Brisbane .. | 3.1 | 48 | $4.3{ }^{\circ}$ | 5.1 | 51 | 51 | 7.0 | 7.2 | 6.0 | 79 |
| Toowcomba | 2.8 | 4.5 | 3.7 | 4.9 | 4.9 | 4.9 | 6.1 | 6.1 | 5.8 | 0.3 |
| Rockhampt'n | 39 | 53 | 40 | 5.4 | 5.3 | 5.2 | 7.3 | 73 | 6.4 | 74 |
| Charters Towers | 4.0 | 5.6 | 3.9 | 6.0 | 6.0 | 5.3 | 8.0 | 8.0 | 6.9 | 8.0 |
| Warwick . | 4.1 | 5.0 | 4.7 | 5.0 | 5.0 | 4.9 | 7.0 | 7.0 | 7.0 | 7.0 |
| Adelaide | 3.6 | 45 | 3.4 | 5.1 | 5.3 | 4.3 | 7.1 | 7.9 | 7.8 | 8.0 |
| Kadina Moonta \& |  |  |  |  |  |  |  |  |  |  |
| Wallaroo .. | 3.5 | 42 | 33 | 4.5 | 5.4 | 4.3 | 0.5 | 6.5 | 61 | 6.8 |
| Port Pirie . | 4.1 | 4.5 | 42 | 6.7 | 59 | 53 | 6.2 | 6.1 | 6.1 | 6.4 |
| Mt. Gambier | 4.0 | 4.6 | 3.5 | 49 | 50 | 4.4 | 6.0 | 6.1 | 6.1 | 6.1 |
| Petersburg | 4 I | 4.7 | 8.7 | 54 | 5.8 | 4.7 | 60 | 60 | 5.8 | 6.0 |
| Perth and Fremantle | 6.7 | 7.4 | 5.9 | 8.0 | 8.1 | 67 | 80 | 9.0 | 8.1 | 90 |
| Kaigoorlie \& Boulder | 7.3 | 8.4 | 7.0 | 87 | 00 | 8.3 | 10.7 | 10.5 | 9.2 | 11.6 |
| Mid Junction \& Guldford | 7.3 | 7.5 | 5.9 | 8.4 | 8.4 | 7.0 | 8.7 | 8.7 | 8.0 | 8.9 |
| Butbury .. | 8.0 | 8.7 | 7.1 | 9.0 | 8.4 | 7.8 | 9.0 | 9.0 | 8.0 | 8.4 |
| Geraldton ${ }^{\text {- }}$ | 6.6 | 6.8 | 5.8 | 76 | 7.6 | 6.9 | 8.0 | 8.0 | 65 | 80 |
| Hobart ${ }^{\text {a }}$ | 4.4 | 5.1 | 4.0 | 6.1 | 6.0 | 48 | 6.2 | 6.5 | 6.6 | 7.0 |
| Launceston ${ }^{+}$ | 4.8 | 5.1 | 3.9 | 6.0 | 5.9 | 5.0 | 6.1 | 6.3 | 6.3 | 6.5 |
| Zeehan . | 6.0 | 6.8 | 5.7 | 6.9 | 7.0 | 6.1 | 7.5 | 7.5 | 7.1 | 7.8 |
| Beaconsfleld | 5.1 | 5.3 | 4.2 | 0.2 | 6.1 | 5.9 | 6.2 | 6.2 | 6.2 | 6.5 |
| - Queenstown | 6.3 | 6.3 | 5.6 | 6.9 | 7.0 | 6.3 | 7.0 | 7.0 | 7.0 | 7.1 |
| Weighted ${ }_{\text {Average }}$ | 3.8 | - 4.7 | 3.7 | 5.5 | 5.2 | 4.3 | 7.2 | 7.7 | 7.2 | 8.0 |

[^10]
## APPENDIX IV.

Weekly House Rents $\dagger$ in Metropolitan Towns, 1901 to 1912.*


* For the first 9 montlis. $\quad \dagger$ The rents are shewn to the nearest penny.

APPENDIX V .

Current Weekly House Rents $\dagger$ in Metropolitan and Country Towns, 1912.*


* First 9 months only. $f$ The rents are shewn to the nearest penny,


| commoditx. | Umix. | 1877 | 1878. | 1879. | 1880. | 1881. | 1882. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. Me. |  |  |  |  |  |  |  |
| - Pig Mix | ton | 102 | 87 190 19 11 |  | 102 211 | $\begin{array}{rrr}91 & 5 \\ 180 \\ & 11\end{array}$ |  |
| Angle and Tee | ", | 204 | 1985 | 1985 | ${ }^{2318}$ | $1{ }^{197}{ }^{19}$ | 201 |
| ${ }_{\text {Plate }}^{\text {Hoop }}$. ${ }^{\text {a }}$ | ", | 242 | ${ }^{221}{ }^{205}{ }^{64}$ | ${ }^{2064} 9$ | ${ }_{240}^{248}$ | ${ }_{212}^{210}{ }_{21}^{1 / 4}$ | 202 |
| Gavanised corrugated | " | 539 | 46810 | 451 | 484 | ${ }^{435} 184$ | 43810 |
| Zinc-Sheet Fencing | ", | - 307 | 285 | 268 507 50 | 283 <br> 540 <br> 1 | ${ }_{530}^{283}$ |  |
| Iead-Shieet | ", | ¢800 | 5231 | ${ }_{477} 4$ | ${ }_{4} 7611$ | 468 | 460 |
| Copper-Sheet. | tib. |  | $\begin{array}{cc}560 \\ 1 & 18 \\ 18\end{array}$ | ${ }^{560}{ }_{1}{ }_{0}^{0}$ | 560 1 1 0 | ${ }_{0}^{544}{ }_{11}{ }^{7}$ |  |
| coapert (on Whari) | ton | 31.2 | 305 | 29 3 | 30104 | 25117 | ${ }_{30} 7$ |
| Jute Goods-Branbags | doze |  |  | ${ }_{6}^{614}$ |  |  |  |
| Cornsacks $\begin{gathered}\text { Woolpacks }\end{gathered}$ | each | 9 5 <br> 3 5 | ${ }^{9} 100$ | ${ }_{2}^{7} 104$ | ${ }_{8}^{8} 9$ | 9 9 3 3 3 |  |
| Leather-Eip ${ }^{\text {Calf }}$ | ${ }^{\text {lb }}$. | ${ }_{8} 1107$ | ${ }^{1}$ | . |  |  |  |
| $\underset{\text { Casils }}{\text { Caif }}$ | dozzen |  |  |  |  |  |  |
| Cotton-Raw $\because$ | lb. | ${ }^{0} 68$ | ${ }_{0} 6$ |  | 0 0 | ${ }^{0} 6$ |  |
| Sool ${ }_{\text {Sll }}$ | " | 20 1 1 0 | $\begin{array}{cc}17 & 7 \\ 17 \\ 1 & 2 i\end{array}$ |  | ${ }_{1}^{17} 10$ | (16 118 |  |
|  |  |  |  |  |  |  |  |
| Wheat | bushel | ${ }^{687}{ }^{63}$ |  | -5 ${ }^{5}$ | - ${ }^{4} 8$ | ${ }^{4} 8$ | ${ }^{6}{ }^{16}$ |
|  | bushel | ${ }^{287}{ }_{1}{ }^{4}$ | ${ }^{2} 181$ | 226 ${ }^{1}$ | 1808109. | $2{ }^{1} 10$ |  |
| ${ }^{\text {Pollard }}$ Ofst. $\quad .$. | " | $\begin{array}{ll}1 \\ 3 & 4 \\ 88 \\ 8\end{array}$ | 1 4 <br> 4 4 | 1  <br> $\frac{1}{3}$ 38 <br> 38  <br> 8  | ${ }_{2}^{9} 114$ | $\begin{array}{ll}1 & 14 \\ 3 & 1 \\ 0\end{array}$ | ${ }_{3}^{1851}$ |
| Oatmeal $\because$ | tos, |  | 575 | ${ }_{466}{ }^{2}$ | 344 | 33810 | 878 |
| Barley - Mallting | bushel | (1) | 6 4 4 4 18 18 |  | - ${ }_{2}^{4}$ | ${ }_{2}^{3} 80$ | 48 <br> 4 <br> 4 <br> 8 |
| Maize |  | 438 |  | 371 | ${ }_{3}{ }^{2}$ Ot |  |  |
| Hay ${ }^{\text {Straw }}$. | ton | 115 0 <br> 67  <br> 6  | ${ }^{102} 818$ | 119  <br> 69  <br> 8  | 87 11 <br> 50  <br> 50  | ${ }_{45}^{83}$ |  |
| ${ }_{\text {Preas }}$ | bushel | ${ }^{3} 1{ }^{3} 1 z^{3}$ | $\begin{array}{lll}5 \\ 5 \\ 9 & 3 \\ 9 & 7\end{array}$ | ${ }^{3} 1074$ | cr ${ }^{2}$ |  |  |
| Growr iv. dairy prödtoe |  |  |  |  |  |  |  |
| Ham.. | lb . | 17 | 0114 |  |  |  |  |
| $\underset{\text { Cheese }}{\text { Bacon }}$ | ", | ${ }^{0} 8818$ | ${ }_{0}^{0} 8$ | ${ }_{0}^{8} 8$ | ${ }_{0}{ }^{0} 5$ |  |  |
| Butter | " |  | ${ }_{0}^{0} 118$ |  |  |  |  |
| Lewd .: | dozen |  |  |  |  |  |  |
|  | 1 l . |  |  |  |  |  |  |
| Currants .. | lb. | ${ }_{0}^{0} 5$ |  | $0{ }^{0}$ |  |  |  |
| Raisuls Herrings Ren |  | ${ }^{8}$ |  | 061 |  |  |  |
|  | tins |  |  | ${ }^{8104}$ |  |  |  |
|  |  | 1210 | ${ }_{8}^{9} 17$ |  | ${ }^{9} 8$ |  |  |
| $\underset{\text { Tea }}{\text { Ceate }}$ S | dozi |  | ${ }^{8} 86$ | 17 |  | 18 |  |
| Coffee $\begin{aligned} & \text { Cocoa } \\ & \end{aligned}$ | " |  | ${ }_{1}^{1} \quad 4$ | $\begin{array}{ll}1 & 3 \\ 1 & 3 \\ 3\end{array}$ | ${ }_{1}^{1} 3$ | $\begin{array}{ll}1 \\ 1 & 17 \\ 3 \\ 3\end{array}$ |  |
| Sugar .. | ¢0n | 780 |  |  |  |  |  |
|  | Ib. | ${ }_{0}^{0}{ }_{0}^{102}$ | ${ }^{0} 9$ | (1) $\begin{aligned} & 0 \\ & 0 \\ & 0\end{aligned}$ | ${ }^{0}{ }_{0}^{011}$ | (1) $\begin{array}{ll}0 & 10 \\ 0 & 28\end{array}$ | ${ }_{0}^{0}{ }_{0}^{10}$ |
|  | ton | $40610^{2}$ |  |  |  |  |  |
| Salt-Fine Rock.. |  | $\begin{aligned} & 901 \\ & 7410 \\ & 70 \end{aligned}$ | 808 58 58 | 82 50 50 | 91 <br> 81 <br> 82 <br> 8 | 84 <br> 868 <br> 68 <br> 6 | 81 48 48 |
| Mustard | 2.11 |  |  |  |  |  |  |
| Starch | $\underset{\substack{\text { tins } \\ \text { lib }}}{ }$ | $\begin{array}{rrr}17 & 5 \\ 0 & 58 \\ 51\end{array}$ |  | 17117 |  |  |  |
| Blue ${ }^{\text {B }}$. |  |  | 011 | ${ }^{6} 10$ |  | ${ }^{0} 91$ |  |
| $\underset{\substack{\text { Masches } \\ \text { Candles } \\ \text { M }}}{ } \quad \therefore$ |  | ${ }^{3} 818$ |  | 3 ${ }^{3}$ |  | ${ }^{2}{ }_{0}^{112}$ |  |
| Kerosene | gallon |  |  |  |  |  |  |
| ${ }_{\text {GROUP }}^{\text {Tobace }}$ VI, M | lb. | 37 | 34 | ${ }^{3} 3$ | 33 | 431 |  |
| Befi. | 100 | . |  |  |  |  |  |
| Mution | ${ }_{\text {ench }} \mathrm{lb}$. | $\cdots$ | $\because$ |  | $\cdots$ |  |  |
| yeal .. $\because$ Pe.. | lb . |  |  |  |  |  |  |
| Pork Hopl Vit. buiidenai | " |  |  |  |  |  |  |
|  | 1000t. lin |  |  |  |  |  |  |
| ¢ ${ }^{6} \times$ | 1 | ${ }_{9}^{10} 9$ | ${ }^{0}$ | ${ }^{8} 8.4$ | ${ }_{7} 111$ | $\begin{array}{r}911 \\ 84 \\ \hline 1\end{array}$ | ${ }_{7}^{10} 0$ |
|  | " | 7 7 6 6 1 |  |  |  |  |  |
| leam | 1000it sp |  | 125102 |  |  | ${ }_{183} 111^{3}$ |  |
| Cement Shelving : |  |  |  | ${ }_{174}^{17}{ }_{17}^{11}$ |  |  |  |
| White Lead | ton | 855 | 7893 | 762 \% | 750 | 735 | ${ }^{14} 78$ |
| Roup Vili. Che |  |  |  |  |  |  |  |
| Carbonate of Soda | ton | ${ }_{30}{ }^{1}{ }_{1}^{4}$ |  |  |  |  |  |
| Saltpetre Sulphur |  | 64310 <br> 958 <br> 8 | 641 279 27 | 658 <br> 278 <br> 8 | B75 3 360 | (808 |  |
| Sulphur | , |  |  |  |  |  |  |

Averağ Annual Wholesale Prices in Melbourne， 1871 to 1912－contd．

| Commodity． | UNIT． | 1883. | 1884. | 1885. | 1886. | 1887． | 1888. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group I．Metals－ |  | s．d． | s．d． | s．d． | s．d． | s．${ }^{\text {d．}}$ | s．d． |
| Iron－Pig Mixed Nos． | ton | 843 | 7810 | 75101 | 724 | ${ }^{7} 7$ | 864 |
| Rod and Bar | ＂ | 1826 | 180 | 16510 | 151 6t | 13988 | 1485 |
| Angle and Tee | ＂， | 1976 | 1990 | 1826 | 164 7t | 1600 | 1531 |
| Plate．．．． | ＂ | 2093 | 210 | 2023 | 180 | 180 | 180 |
| Hoop | ＂ | 2009 | 2000 | 190 O | 180 | 176 | 1712 |
| Galvanised Corrugated | ＂ | 438 | 3995 | 3638 | 3338 | 3383 | 3636 |
| Wire，Fencing ．． | ＂ | 254 | 2545 | 211 61 | －185 4t | 17310 | 1883 |
| Zinc－Sheet | ＂ | 469 24 | 486 | 44611 | 43310 | 4150 | 4274 |
| Lend－Sheet | ＂ | 4443 | 37210 | 35973 | 3969 | 40471 | 41611 |
| Prping |  | $426{ }^{6}$ | 4100 | 4100 | 410 | 371 6i | 4100 |
| Copper－Sheet | 16. | $010 \frac{8}{8}$ | 0102 | 010 | 08 | $\begin{array}{ll}0 & 8 \frac{1}{8}\end{array}$ | 0 11\％ |
| Coal（on Whari） | ton | 289 | 28 6t | 300 | 29 3 | $25 \quad 9$ | 29113 |
| Groupli，Textiles，Leather，de |  |  |  |  |  |  |  |
| Jute Goods－Branbags $\quad \cdots$ | dozen | $4{ }^{4} \mathbf{1 0 4}$ | $\begin{array}{ll}5 & 12 \\ 6 & 12\end{array}$ | $\begin{array}{ll}4 & 14 \\ 5 & 8 \\ 8\end{array}$ | $\begin{array}{ll}3109 \\ 4 & 114\end{array}$ | 3110 | $\begin{array}{ll}4 & 74 \\ 6 & 3\end{array}$ |
| Woolpacks ．${ }^{\text {－}}$ | eiach | 208 |  | 234 | $25 t$ | 28 | $2{ }^{2}$ |
| Leather－Kip ．． | ］b． | $1{ }^{1} 6$ | $1{ }^{1} 6$ | 186 | $\begin{array}{ll}1 & 6 \\ 2 & 6\end{array}$ | $\begin{array}{ll}1 & 6 \\ 2\end{array}$ | ${ }_{6}^{1}{ }^{6}$ |
| Calt ． |  | 28 | 278 | $2{ }^{2} 7$ | 267 | 26 | 26 |
| Basils | doze | 140 | 140 | 140 | 140 | 1403 | 140 |
| Cotton－Raw | lb． | 0 6t | 0 6t | 0 6 ${ }^{1}$ | 0 54 | 0 5 | 0 54 |
| Silk－Raw | ＂ | $16 \quad 2 \frac{1}{2}$ | $149 \frac{1}{2}$ | 14 1． | 13 8 | 137 | 12 3t |
| Wool | ＂ | 105 | $10 \frac{1}{3}$ | 0 104 | 098 | 0 109 | 0108 |
| Grour III．Agriodlic＇P Ptoddoe |  |  |  |  |  |  |  |
| Wheat | bushel | ${ }_{20}^{411}$ |  | $15010{ }^{3}$ | 4 9 9 ${ }^{\text {a }}$ | ${ }^{3} 69$ 92 | 389 |
| Ftour． | ton | 2049 | 17711 | $159{ }^{4}$ | 2037 | 1694 | 1768 |
| Bran． | bushel | 1 1 | $1{ }^{1} 0$ | $1{ }^{1} 0$ | 1.17 | $\begin{array}{ll}0 & 91 \\ 0 & 9\end{array}$ | ${ }_{0} 104$ |
| Pollat ${ }^{\text {d }}$ | ＂ | 1 1 1 | 101 | $10 \pm$ | $1{ }^{1} 1 \frac{1}{4}$ | 09 | 0 9t |
| Oists |  | $3{ }^{13}$ | 308 | ${ }^{2} 10 \frac{1}{5}$ | 210 | 278 | 2103 |
| Ontmeal | ton | 3579 | 3448 | 36310 | 385 | 3766 | 3836 |
| Barley－Maltang | bushel | $5{ }^{5}$ | $4{ }^{2}$ | $4{ }^{4} 4$ | ${ }_{3} 3$ 11t | 5 3t | 55 |
| Feed | n | 310 | 2 11妾 | 2114 | 211 | 37 | 3 2a |
| Maize |  | $4{ }^{4}$ | 5 0t | $4{ }^{2}$ | 4 31 | 4 0f | 378 |
| Hay，． | ton | 12010 | 1042 | 119 － | 1197 | 1263 | 11211 |
| Straw |  | 576 | 58 | 58 | 592 | 618 | 550 |
| Peas ． | bushel | $3{ }^{3} 7$ | 3 4t | 337 | 35 | $314 \pm$ | 2118 |
| Potatoes | toll | $63 \quad 4$ | $75 \quad 5$ | 627 | 71 6it | 61 5t | 70 0 |
|  | lb． |  |  | 0 96 |  | 0 01 | 0 0） |
| Bacon | ＂ | 081 | 088 | 088 | 081 | 088 | 0 |
| Cheese | ＂ | 0 7 ${ }^{\text {¢ }}$ | 0 6 | 078 | 07 | 0 63 | 0 |
| Butter | ＂ | $10 \frac{1}{2}$ | 1 1娄 | $14 \frac{1}{3}$ | 13 \％ | 10 | 1 络 |
| Jard ． |  | 088 | 07 | 07 | 0 6 | 061 | 0 5t |
| Egge ． | dozen | 128 | 1 2 | 138 | 1.2 | 128 | 12 |
| Honey | tb． | 054 |  | 04 | 083 | 0 4t | 0 4t |
| noup Y．Groos Currants | lb， | 51 |  | $4 \frac{1}{6}$ |  |  | 4． |
| Raisins |  | 067 | 0 6t | 0 5t | 05 | 0 5 | 0 詸 |
| Herrings $\quad$ ． | $\underset{\operatorname{cins}}{\text { doz. } 11} 1 \mathrm{lb} .$ |  |  |  |  |  |  |
| Salmon |  | $811 \frac{8}{8}$ | 8 18 | 74 | 891 | 93 | 910 \％ |
| Sardines | doz．Hivs | 98 t | 8 线 | $.710{ }^{\text {d }}$ | $710 \frac{1}{1}$ | 79 | 714 |
| T＇ea． | lb． | 1．34 | 12 | $13 \frac{1}{8}$ | $12 \frac{3}{2}$ | 1 18 | 12 |
| Cotfee | $\because$ | $1{ }^{1}$ | 108 | 10 | $011 \%$ | 18 | 13 |
| Cocoa |  | $13 \%$ | 13 | 1 27 | 1 2t | 123 | 1 23 |
| Songar | ton | 7550 | 6837 | 518 6 | 4943 | 4859 | 4709 |
| Macarol | 1 l ． | 0 10t | 088 | 074 | 073 | 0 7 | 0 71 |
| Sago |  | 0 2t | 0 1音 | 0 11 | 0 11 | 0 13 | 021 |
| Tilice | ton | 43610 | \＄59 0 | 4389 | $406 \cdot 6$ | 4400 | 413 0 |
| Salt－Fiat ${ }^{\text {c }}$ |  | 807 | $80 \quad 9$ | 819 | ． 847 | 818 | 885 |
| Rock |  | 527 | 58 0 | $53 \quad 3$ | 088 | 65 6 | 559 |
| Milustard | $\left\|\begin{array}{c} \mathrm{doz}, 1 \mathrm{lb} \\ \operatorname{tins} \end{array}\right\|$ | 18 34 | 18 | 18 3 |  |  | 17 918 |
| Starch | lb． | 0 96 | 051 | 05 | 0 4 | 0 4 ${ }^{\text {a }}$ | 0 4 |
| Blue ．． |  | 0 018 | 0 9 | 091 | 0 8． | 08 | 081 |
| Matches | gross | 111 | 22 l | 244 | 22 2 | 191 | 19 |
| Caudles | lb． | 0 0 ${ }^{\text {a }}$ | 0 9t | 088 | 071 | 06 | 06 |
| Kerobene | gailon | 158 | 1 51 | 14 | 15 | 1 6 | 17 |
| Tobacco | lb． | 45 | 45 | 4 4 | 45 | 45 | 4 4 |
| Group VI．Meat－ |  |  |  |  |  |  |  |
| Beef ．．． | 100 lb | ． | 23 3年 | 21 71 |  |  |  |
| Mutton | 1 b ． | ． | 021 | 0 1亲 |  |  |  |
| Lamb | each |  | 7 2t | 7 4 |  |  |  |
| Veal | ib． |  | 0 2 | 031 |  |  |  |
| Pork |  |  | 0 7t | 063 |  |  |  |
| Group Vit．Bullding material Timber－Flooring－ $8 \times 11$ |  |  |  |  |  |  |  |
| $\text { Timber-Flooting-6 } \times{ }_{6}^{6} \times \frac{1}{6}$ | $100 \mathrm{ft}$. lin | $\begin{array}{ll}9 & 94 \\ 7 & 108\end{array}$ | 8 8 5 5 |  | $\begin{array}{ll}8 & 105 \\ 7 & 1\end{array}$ | $\begin{array}{ll}8 & 7 \\ 6 & 8\end{array}$ | $\begin{array}{ll}9 & 01 \\ 8 & 1\end{array}$ |
| $6 \times$ x | ＂ | 67 | 5 5 $2 \frac{1}{2}$ | 6 硣 | 510 \％ | $5{ }^{5} 581$ | 6103 |
| $6 x$ |  | 50 0 | 46 | 541 | 4 64 | 43 | 468 |
| Weatherboards |  | 604 | 6 04 | $5{ }^{6}$ | 4101 | 41.02 | 6 0t |
| Oregon ． | 1000ft sp | 1360 | 12910 | 1378 | 081. | 1113 | 1381 |
| Shelving |  | 2273 | 222118 | 218 6年 | 1938 | 223 | 250 |
| Cement＋${ }^{+}$ | cask | 15 2t | 1508 | 142 | 123 | 121 | 15 88 |
| White Lead | ton | 6347 | 6086 | 5500 | 5309 | 5350 | 5986 |
| Group Vili．Chemioals－ Cream of Tartar | lb． | 1 4 ${ }^{\text {娄 }}$ | 1.48 | 1 4t | 1 | 1 4혈 | 47 |
| Carbonate of Soda | ton | 2384 | 21310 | 2046 | 195 | 188 4 | 166 |
| Saltpetre | ＂ | 6.60 | 6515 | 6300 | 57610 | 5647 | 5478 |
| Sulphar－ | ＂ | $260 \quad 0$ | 2600 | 2447 | 2362 | 2186 | 2347 |

xvill.
Average Annual Wholesale Prices in Melbourne, 1871 to 1912 -contd.


Average Annual Wholesale Prices in Melbourne, 1871 to 1912-contd.


Average Annual Wholesale Prices in Melbourne, 1871 to 1912-contd.





## Appendix



[^11] Lhe price of any mikle noce the medide of wet month).
(Continue rambike on bect al aboek, it necersery )



Toum


RETAIL PRICES.




(Continue romarts on buck of aboek, il oectusary)

Drstract or Swhoth_一 Ofice No R P $t 6$ — WEEKLY HOUSE RENTS


| Name of Apritich addross_ |  |  |  |  |  |  |
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| Thise olip in to bo dalecind and pooted ta the ' Comanomentich Statratichil Melboutre $\qquad$ <br>  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| $\begin{aligned} & \text { Nasure } \\ & \text { How } \end{aligned}$ |  | + + +oome | - Eavera | + Reomes | \% Mowers |  |
| Fend <br> Beint *q |  |  |  |  | $12$ |  |
| ESMARKY - tranes bane the caume of anty alranes of dipling is reats ulwot tho preseding quartorfin. |  |  |  |  |  |  |

## APPENDIX VIII.

# THEORY OF DETERMINING PRICE-INDEXES SHEWING VARIATIONS IN THE EXCHANGE-VALUE OF GOLD, OR IN THE COST OF LIVING. 

BY G. H. KNibBS, C.M.G., F.R.A.S., F.S.S., Etc., Etc.<br>Federal Statistician, Australia.


#### Abstract

\section*{SYNOPSIS.} 1. General theory of determining price-indexes. 2. Price-indexes from relative total expenditures and from price-ratios. 3. Arithmetic, geometric, and harmonic means. 4. The harmonic mean is really as legitimate as the arithmetic, but is not more so. 5. Weights to be applied when price-ratios are used. 6. Computations of mean wetghts. 7. Error of means. 8. Index-numbers referred to average conditions during a period. 9. Differences between various price-indexes. 10. Varous mothods adopted for measuring the exchange-value of money. 11. Supposed defects in the geometric mean. 12. The aggregate expenditure method the best. 13. Conclusion.


1. General Theory of Determining Price-Indexes.-The exchange-value or purchasing-effaciency of money 18 measured by the amount of any commodity which a unit of money ( $\mathbf{x} 1$ say) will purchase ; or it is measured in a reciprocal way by the amount of money or price which has to be pard for a unit of the commodity in question. The lattor measure is, of course, a reciprocal of the former, that is, the exchange-value or money-purchasing efficsency of the commodity is measured by the quantity of money which a unit of the commodity will buy, or for which it can be exchanged. It is convement, and is the custom, to express exchange-values through price. When the price of a commodity changes (for example, when it becomes greater) it denotes change of (reduced) efficiency in the purchasing-power or exchange-value of money with respect to that commodity. Thus af a thing that originally cost $£ 4$, at somelater date costs $£ 5$, the price has aduanced in the ratio from 1 to $\frac{9}{4}$ or $25 \%$, or the efficiency of the purchasing-power or exchange-value of money has, in respect of the commodity in question, fallen from 1 to $\frac{3}{3}$, or $20 \%$, the two statements being virtually the same. The ratio of the price at one date to that at another is called ats price-ratio in respect of those dates. It has become customary for economists to regard every instance of a rise or fall in price in a particular commodity as an individual measure of a variation on the exchange-value of money, a measur's which has value or weight in proportion as expenditure upon the commodity in question enters into the aggregate of expenditure upon the whole series of commodities of which it forms a single member. The term "exchange-value" is to be preferred because it is unambiguous; "value" without qualification might denote utility-value, esteem-value, cost-value, etc. Here it may be remarked that the method of determinug variations of the exchange-value of gold by means of priceratios, is not a good one, as is shown hereinafter, and the only satisfactory method is that of aggregate expenditures for a given regimen.

## APPENDIX.

Now it is obvious that if, in a series of commodities, the quantity used in a given period be constant for each commodity, the measure of the economs importance or economic "weight" of each is the relative expenditure m money units on that member of the series.* Hence through statistics we may obtain some idea of this massure or "weight." Werght in this sense has, of course, no direct connection with physieal weight.

When prices have changed, however, the "weights" will have changod also, unless the quantities of the commodities have changed so as to leave the expenditures (or quantities multiplied by the price) the same. Ordmarily it may of courso be said that the "weights" will have changed. Now there can be no real comparison of the relative purchasing-power or exchange-value of money, excopt on some supposition of constancy in human requirements, and just in proportion as the usage of different commodities varies so will any estimate of relatave purchasing. power become dubious. In short, a fixed regimen is essential for an accurate determination.

In some instances human requirements are iairly constant. If we suppose thet, for an "average" member of the community, a partacular regimen be adhered to, then clearly we may tabulate the aggregate expenditure on that regmen at two dates; and the expenditure thereon at the later date, divided by the expenditure at the former, will measure the expenduture-ratio for the two dates. 'Thus, for example, if we suppose it to increase, it will represent a rise in the cost of the commodities. The reciprocal of this ratio or relative increase measures the decrease in the purchasing-power of money with respect to the particular regimen.

If the regimen itself vary, any computation of the change in question is dubious, because it contains two elements, viz. :-
(i.) Change in the regimen itself, i.e., change in the use of the commodities (or standard of living), and
(ii.) Change in the oxpenditure on the cost of the individual elements in the regamen.
Where the regimen changes either in virtue of the changes in price, arbatrarly, or in response to changes in the "standard of living." etc., there are still assumptions by means of which accurate comparisons can be made. This we may make several definite suppositions, for example :-(a) that the quantities at the former date apply to the later, and thus compute what the effect of changed price would be; or (b) we may, on the other hand, suppose that the quantities used at the second date were actually those at the earlier date, and can again compute the aggregato cost of the regimen on this assumption. Both of these comparisons are, in their way, valid, intelligible, and respond to certain questions of sociological importance that from time to time arise, and which for certain purposes domend an answer. The best general assumption (c) is, of course, that some mean-value of the "weights "apphes: this mean may be arithmetic, geometric, or harmonic ; and any one of these means may naturally arise. It is shewn hereinafter that the geometric mean is doulbtless the most accurate generally, but that in certan cases the arithmetse may be used.

If we have price-ratios for a series of commadities, and deduce from them some general ratio that expresses for the series in question the price on the whole at the second date, as compared with the former, such a ratio is called the price-index of the latter date.

The nature of the combination of the price-ratios in the calculation of a priceindex, even when the relative weights are decided, is a matter for consideration. It is essential, for example, for satisfactory comparisons that a serics of priceindexes which profess to express changes in the purchasing-power of money shall furnish the same relation between the purchasing-power at any two dates, as would be furnished by calculating by the method approved from the original data for the two dates. If this were not se, then obvously the undex-numbers do not fulfil their profession; in short, they are misleading.

Index-values, as ordinarily furmshed, are unfortunately subject to this criticism, viz., that they cannot, in the nature of the case, be assumed to represent intelligibly the relation required, at least with suffictent precision to answer many practical questions. This may be readily seen by comparmg any two seriss of price-indexes.
2. Price-Inderes from Relative Total Bxponditures and from Price-Ratios.-For a series of commodities A, B, C, the price at a certain date is $a_{0}, b_{0}, c_{0}$, etc. ; at some later date it is $a_{1}, b_{1}, c_{1}$, etc. The quantitics of these commodities may be denoted by $a, \beta, \gamma$, ete., with suffixes 0 and 1 according to the date. The unit by which any commodity A, B, or C, etc., is measured may of course be anything whatever,

[^12]as a pound avoirdupos, a gallon, a gross, an article, etc. The price-ratio at the latter date as compared with the former ordnarily varies with the commodity used to determine it. Thus it may be written :-
\[

$$
\begin{equation*}
\cdots \cdots{ }_{0} p_{1}=\frac{a_{1}}{a_{0}} ;{ }_{0} q_{1}=\frac{b_{1}}{b_{0}} ;{ }_{0} r_{1}=\frac{c_{1}}{c_{0}} ; \text { etc., } \tag{1}
\end{equation*}
$$

\]

according as commodity $A, B$, or $C$, etc., is used. In attempting to utilise these price-ratios $p, \boldsymbol{q}, r$, etic., for any general deduction, the relative-weight which should be ascribed to each obviously demands consideration. If the quantities or mass-units usod wore identical at each date, and were, say, a, $\beta, \gamma$, etc., we should have a general price-ratio $I$, determined as follows, viz. :-

$$
\begin{equation*}
\cdots \cdots{ }_{0} I_{1}=\frac{P_{1}}{P_{0}}=\frac{a b_{1}+\beta b_{1}+\gamma c_{1}+\text { etc. }}{a a_{0}+\beta b_{0}+\gamma c_{0}+\text { etc. }} \tag{2}
\end{equation*}
$$

$P_{0}$ and $P_{1}$, denoting total expenditure on commodities $A, B, C$, etc., and ${ }_{0} I_{1}$ the deduced general price-index for the dates in question. This formula is one which, for example, would give the relatuve cost of living at the two dates, on the assumption that the cominodities, A, B. C, ete., represented the standard of living, and that the quantities of them wheh wero consumed were identical at the two dates. The formula given (2) is unquestionably the only proper formula to use in such a case, and it may be shewn that it is the best formula in all cases.' See hereinafter.

To clearly illustrate the matter, suppose, for example, that at the two dates this consumption for some given unit of time was, let us say, uniformly ten $4-1 \mathrm{lb}$. loaves, 1 pound of tea, and 5 quarts of milk.* Suppose further that the prices at date 0 wore respectively 5 d . per loaf, 15d. per 1 b ., and 6d. per quart, and at date $1 \mathrm{6d}$. per lonf, 24 d . per 1 lb ., and 4 d . per quart. Then on this assumption the actual cost of Inving (so far as these items are concerned) would have risen from 95̃d. to 104d, that is in the ratio of 100 to 109.47 , or in other words, there would be a $9.47 \%$ increase in the " cost of living." $\dagger$

A method very commonly employed, however, for estimating changes of this kind is to ascertain the price-rutio for each commodity, that is, to find the quantities $p, q, r$, etc., by dividing the price per unit at the second date by that at the first, and to take a mean of all for a general result. $\ddagger$ The price-ratio is, of course, independent of the size of the unit. These guotients are commonly multiplied by 100 for mere convenience.

As reliance upon price-ratios and combinations of them is very common. the question will be referred to at some length.

If price-ratios were really of equal weight we should have

$$
\begin{align*}
& (3) \ldots I^{\prime}=\frac{1}{n}(p+q+r+\text { etc. to } n \text { terms }) ; \text { or } \\
& (4) \ldots I^{\prime \prime}=v^{i}(p . q . r . \text { etc. to } n \text { terms }) ; \text { or } \\
& (5) \ldots . I^{\prime \prime \prime}=\frac{1}{\frac{l}{n}\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{r}+\text { etc. }\right)}=\frac{n(p . g . r . \text { etc. })}{(q r \ldots)+(p r \ldots)+(p q+)+\text { ete. }} \tag{5}
\end{align*}
$$

according as we preferred the arithmetic, the geometric, or the harmonic mean. Which of these is to be preferred is a point to which we shall refer later. The arithmetic, geometric and harmonic means all assume that each commodity is of equal importance in the result, but which is the proper one to adopt depends on other considerations of a more complex character. Popularly the arithmetic mean, viz., the ordinary average, is supposed to be satisfactory, but this is an error arısing ordinarily from the fact that what underlies such an assumption is not apparent. Taking

[^13]the example just quoted, and regarding the evidence of each commodity as to rise of price as of equal value, we should have the following results according as we take one or the other mesn, viz. :-

Arithmetic Mean.

Sum
3.4667
(Divisor 3.)

100 $\times$ Quotient, 115.56

Geometric Mean.
1.20 •

1. 60
0.6667

Product 1.280

- (Root, Cube)
$100 \times$ Result, Cube Root

Harmonic Mean.

|  | $\frac{5}{6}$ |
| :---: | :---: |
|  | 15 |
|  | $\overline{24}$ |
|  | 6 |
|  | $\overline{4}$ |
|  | 71 |
| Sthm | $\overline{24}$ |

(Divisor 3)
Quotient $\frac{71}{72}$.

Result by ratio of aggregate of expenditures $=109.47$.
Thus we have four results, viz.. by formula (2), viz., the ratio of total expencliture $109.47 \%$; by formula (3) based on the unwerghted arithmetic moan 115.56 ; by formula (4) based on the unweighted geometric mean 108.58 ; and by formuta (5) based on the unweighted harmonte mean 101.41. And it may be added that had we used formula (2) with one unit of each (i.e., $\alpha=\beta=\gamma=1$ ) we should have obtamed the result. $130.77 \%$, or $30.77 \%$ merease, and further that by a mothod given hereinafter we should obtain 109.53.

The illistration shews conclusively that the weight assigned to cach is of great importance, but before dealing with this it is necessary to consider how soveral means can arise in determining price-madexes by means of pricc-ratios.

## 3. Significance of Arithmetic, Geometric, and Harmonic Means of Price-Ratios..-

That there may be different means has niready been referred to. A word is necessary as to their nature. When an inerement to any quantity 18 wiform and independent of the magnitude of the quantity itself, that is, when it is dependent merely upon the interval of time elapsing. and is equal for equal times, then the progression is arithmetic. This is expressed by the following, viz. :-

$$
\text { (6). } \ldots \ldots(a) ; \quad a+\frac{b-a}{2}=\left(\frac{a+b}{2}\right) ; \frac{a+b}{2}+\frac{b-a}{2}=(b) ;
$$

the common difference being $\frac{1}{2}(b-a)$, and the quantity $\frac{1}{2}(a+b)$ being the arithmetic mean of $a$ and $b$. Often, however, in the nature of the case the magnitude of the increase is actually dependent on the magnitude of the quantrty to which it is added; for example, compound interest on money, that is, the rate of increase, is constant : then the progression is geometric, for exsmple :-

$$
\begin{equation*}
(a) ; a \times \sqrt{\frac{b}{a}}=(\sqrt{a b}) ; \quad \sqrt{a b} \times \sqrt{\frac{b}{a}}=(b) \tag{7}
\end{equation*}
$$

the common ratio being $\sqrt{ }(b / a)$ and $\sqrt{ }(a b)$ denoting the geometric mean of $a$ and $b$. We thus see that the square-root of the product of $a$ and $b$ is the mean valuc, when $a$ increases to $b$ at a constant rate on the increasing quantity. Thore is another important way in which a quantity can grow.' Suppose $a$, in the fraction $\frac{1}{a}$, increases (or diminishes) to $b$, in the fraction $\frac{1}{b}$, independently of the magnitude of $a$ (or of $b$ )
Then we have:-

$$
\text { (8) } \ldots \ldots\left(\frac{1}{b}\right) ; \frac{1}{b-\frac{b-a}{2}}=\left(\frac{1}{\frac{1}{2}[a+b]}\right) ; \frac{1}{b-2\left(\frac{b-a}{2}\right)}=\left(\frac{1}{a}\right)
$$

Multiply these by $a b$ and we get :-

$$
\begin{equation*}
(a) ; \quad\left(\frac{2 a b}{a+b}\right) ; \text { and }(b) \tag{9}
\end{equation*}
$$

Then either series of the quantities in the brackets in (8) or (9) are in harmonic progression, formula'(8) giving the form in which the progression arises in question of change in price-ratios, formula $\{9\}$ that which is usually given as the harmonic mean between $a$ and $b$.
4. The Earmonic Hean is really as legitimate as the Arithmetic, but is not more so, and both are invalid.-The question of the legitimacy of employing either the arithmetic or the harmonic or the geometric method of arriving at a price-index may readily be illustrated by means of examples. It may be premised that if, at the beginning of a period of time, the price-index be taken as unity, and at the end of the period it is found by any process to be, say $I$, then, starting at the end of the period with a price-index of unity, and working back by the same process, one should arrive at $l / I$ as the price-index at the beginning. In other words, to have any definite meaning the ratio between the two index-numbers should always be maintained if the scheme of calculation be arithmetcally walid. With this principle as a erucial test, the question arises which, if any, of the three methods of arriving at priceindexes will satisfy the test. Taking the same example as before, where three com. modities, whose starting price is unty, changed in price until they stood respectively at $1,20: 1.60$ : and .6667 or $\frac{8}{6}$, $\frac{8}{5}$, and $\frac{3}{3}$ we have-

$$
\begin{aligned}
& \text { Arithmetic Mean }=\frac{1}{3}\left(\frac{6}{5}+\frac{8}{5}+\frac{2}{3}\right)=1.1556 . \\
& \text { Geometric Mean }=\sqrt[a]{ } /\left\{\frac{6}{5} \cdot \frac{8}{5} \cdot \frac{2}{3}\right\}=1.0858 . \\
& \text { Hammonc Mean }=\frac{3}{\frac{5}{6}+\frac{5}{8}+\frac{3}{2}}=1.0141 .
\end{aligned}
$$

Consequently the new index-numbers are respectively $1.1556,1.0858$, and 1.0141 . The recprocals of these numbers are respectively $0.8654,0.9210$, and 0.9861 . Consequently if the process of obtaining the index-numbers be reversed, and we start at the end of the period, assuming that the corresponding index-number is unity, and then work back to the beginning by the three processes, we ought to find that the arithmetic gives 0.8654 as the price-index at the start, the hermonic process ought to give 0.9861 , and the geometric ought to give 0.9210 . We will see now what actually does happen. Our three price-ratios become $\frac{5}{6}$, 乭 and $\frac{9}{2}$.

$$
\begin{array}{ll}
\text { Arithmetic Mean } & \frac{1}{3}\left(\frac{5}{6}+\frac{5}{8}+\frac{3}{2}\right)=0.9861, \text { instead of } 0.8654 . \\
\text { Geometric Mean } & \sqrt[4]{ }\left\{\frac{5}{6}: \frac{5}{8} \cdot \frac{3}{2}\right\}=0.9210, \text { as before } 0.9210 . \\
\text { Harmonuc Mean }= & \frac{3}{\frac{6}{5}+\frac{8}{5}+\frac{2}{3}}=0.8654, \text { instead of } 0.9861 .
\end{array}
$$

We thus see that the arithmetic process gives the result expected from the harmonic process, and the harmonic, the result expected from the arithmetic; but the geometric process gives' the result expected from that process. That is, neither the arithmetic nor the harmonic process is reversible, and this is a grave defect, in fact a fatal one, as regards their practical use. The geometric process alone satisfies the indicated test of consistency.
5. Weights to be applied when Price-ratios are used.-Attention may now be given to the important question of werghting, if price-ratios are used. It is obvious that relative units of quantities used in the same period must be employed with the method of expenditures ; formula (2). Reverting to the original illustration, we may further consider the case of the three commodities whose prices, starting at

## Appendix.

unity at date 0 , become respectively $1.20: 1.60:$ and 0.6667 at dato 1 . We shall denote the weights by $u, v, w$, ete.; they measure relatively the expenditure on the corresponding commodities. Three courses may be edopted.
(i.) The price-ratios can be weighted according to the respective expenditures at date 0 .*
(ii.) The price-rathos cas be weghted according to respective oxpenditures, at date 1.*
(iii.) The price-ratios can be weighted according to some mean of the two oxporiditures. Of these mean-weights, there are only three which it is at present proposed to consider, viz., those already referred to (a) the arithmetic, ( $b$ ) the geometric, and ( $c$ ) the harmonic.*
These deduced mean-weights (iiia), (iin.b) and (ith.c) can be computed by formula (3), (4), and (5) if we substitute $u, v$, and $w$ for $p, q$, and $r$ therein, and tho different weights, computed in the way indicated, are shewn in the following table:-


The respective index-numbers, computed as, by formula ( 10 ) heremaiter, viz., that which is most commonly used, are given by the amounts

$$
\frac{(50 \times 1.20)+(15 \times 1.60)+(30 \times 0.6667)}{50+15+30}
$$

and four other similar expressions. Ther values multipliod by 100 are :

## Index according to-m

(i.) $=109.47 \dagger$;
(iii. $a$ ) $=114.44$;
$($ iii.c) $=114.41$.
(ii.) $=118.97$;
(iii.b) $=114.43 ;$

The last three results, though worthless, are almost identical, but (iti,b) and (iii.c) would, of course, not be employed with formulæ (3) or (10) herenfter. Given the weights to be adopted, we may now consider the question how the price-index should, be computed if price-ratios aro used at all. We may remark that a " weighted mean" is the mean that would be obtained by regarding each item as repeated the number of times indicated by the weight number.

Let the weights corresponding to commodities $A, B$, and $C$ for to tho priceratios $p, q$, and $r$ ) be denoted by $u$, $v$, and $w$.

Then mstead of formulae (3), (4) and (5), we have, using $J$ (instead of $I$ ) to denote a weighted-mean, the following formulae, aceordmg to whether we employ the arithmetic, geometric, or harmone mean :

$$
\begin{align*}
& \text { (10) } \ldots \ldots . J^{\prime}=\frac{1}{(u+v+w+\text { etc. })}(u p+v q+w r+\text { etc. }) \\
& \text { (11) } \ldots \ldots J^{\prime \prime}=\left\{\boldsymbol{p}^{\prime \prime} \cdot \mathscr{Y}^{v} \cdot \tau^{i v} \text {, etc. }\right\} \frac{2}{u+v+i v+\text { etc. }} \\
& J^{\prime \prime \prime}=\frac{u+v+w+\text { etc. }}{\frac{u}{\nu}+\frac{v}{q}+\frac{w}{r}+\text { etc. } .} \tag{12}
\end{align*}
$$

[^14]By taking logarithms of (11) we see that we obtan a result analogous in form to (10) since

$$
(11 a) \therefore \ldots \log . J^{\prime \prime}=\frac{1}{u+v+w+\text { etc. }}(u \log . p+v \log . q+w \log . r+e t c .)
$$

We soe thus, that using the weights indseated by (ii.b) according to formula (11), we have

$$
\log . J^{\prime \prime}=\frac{1}{98.23},\{54.77 \times 0.07918+18.97 \times 0.20412+24.49 \times \overline{1} .82391\}
$$

thus $J^{\prime \prime}=1.0956 ;$ and $100 \mathrm{~J}^{\prime \prime}=109.56$.*
Lastly, using the weights indicated (II.c) according to formula (12) we get

$$
J^{\prime \prime \prime}=\frac{54.55+18.46+24}{\frac{54.55}{1.2}+\frac{18.46}{1.6}+\frac{24}{0.6667}}=\frac{97.01}{92.99}=1.0432 ; \text { and } 100 J^{\prime \prime \prime}=104,32
$$

From the examples given it will be noticed that when the price-index 18 computed as a geometric mean, it lies between values given by the harmone and arithmetic means, the arithmetic being the highest. Incidentally, it may be remarked that it is obvious that the weighted geometric mean will be lower than the weighted arithmetic mean, since. with numbers groater than unity and very near unity, the difference letween the logarithms of numbers, is much less than the differences between the numbers themselves. Thus, as we see at once from formula (1la), the effect of large dufferences of weighting must necessarily be less when the geometrie mean is computed, rather than the arithmetic. That on other grounds the geometric mean is also to bo preferred can be seen mstantly from the fact that it incidentally gives consistent results in whatever way we work from one date to another, whereas the arithmetic and harmonce means do not give consistent results. By parity of process differences of value may in general be approprately measured by their relation to the quantity which fluctuatos, and this conception of rate-variation necessarily leads to the adoption of geometric means: or to put it in other words, - the moment price-variation is looked at from the standpoint of rate-differences (for example, Id. is $10 \%$ th the case of an article the value of which is 10 d ., but only $5 \%$ where the valite of the article is 20 d .) we see at once that all measurement of change of value may quite appropriately be in rates, and, consequently, the geometric mode of computing may be regarded us the legatimate one where the original data are supplied in the form of prace-ratios. Finally, it may be noted that the weighted geometric mean, the weights being given by ( $11 . b$ ), in conjunction with formula ( 11 ), is 109.56, by (iii.a) and formula (il) is 109.53 , and by the cost-of-living formula, viz. (2), is $109.47 \dagger$; and further if the original weights 50,15 , and 30 for commodities $A, B$, and $C$ be taken, and the weighted geometric mean of the price-ratios be calculated, we obtain 104.30 (practically identical with the harmonic result of formula (12) just given). This shews that it is by no means satisfactory to use the original weights, as is usually done in the case of price-indexes, unless there be reason to believe they are sensibly constant throughout.
6. Computation of Mean Weights.-We now reach the discussion of the general problem of which the example just given is a particular case. Suppose at date 0 the prices of a series of commodities are respectively $a_{0}, b_{0}, c_{0}$, etc. ; and an amount $a_{0}$ is bought of tho first, $\beta_{0}$ of the second, etc.; the total expenditure on the first amounting to $\zeta_{0}$, on the second to $\eta_{0}$, etc. Suppose further that at date 1 the respective prices are $a_{1}, b_{1}$, etc., the price-ratios $\frac{a_{1}}{a_{0}}$, etc., are denoted by $p, q, r$, etc., and the total expenditures by $\zeta_{0}, \eta_{0}$, etc., at date 0 , and $\zeta_{1}, \eta_{1}$, etc., at date 1 . Then weighting the different quantities by the geometric means of the expenditures according to the geometric formula, we have, since

$$
\begin{aligned}
&(13) \ldots \ldots \zeta_{0}=a_{0} a_{0} ; \quad \zeta_{1}=a_{1} a_{1} ; \text { etc. } \\
&(13 a) \ldots \ldots \xi_{0}=\beta_{0} b_{0} ; \quad \eta_{1}=\beta_{1} b_{1} ; \text { etc. }
\end{aligned}
$$

for index-number at date 1 ,

$$
\text { (14) } \ldots \ldots J^{\prime \prime}=\left(p^{\sqrt{\zeta_{0} \zeta_{1}}} \cdot q^{\sqrt{\eta_{0} \eta_{1}}} . \text { etc. }\right)^{\frac{\sqrt{\zeta_{0} \zeta_{1}}+\sqrt{\eta_{0} \eta_{1}}+\text { eto. }}{}}
$$

[^15]a. number whose logarithm is
$$
(15) \ldots \ldots \log . J^{\prime \prime}=\frac{\sqrt{\xi_{0} \zeta_{1}} \log . p+\sqrt{\eta_{0} \eta_{1}} \log . q+\text { etc. }}{\sqrt{5_{0} \zeta_{1}}+\sqrt{\eta_{0} \eta_{1}}+\text { etc. }} .
$$

Consequently when the total expenditures at any two poriods are at all comparable, we may put with sufficient accuracy

$$
\text { (16) } \ldots \ldots \zeta \zeta=\frac{1}{2}\left(\zeta_{0}+\zeta_{1}\right) ; \text { and } \sqrt{\zeta_{0} \zeta_{1}}=\zeta-\frac{1}{8} \frac{\left(\zeta_{1}-\zeta_{0}\right)^{2}}{\zeta}+\text { etc. }
$$

The term - $\left(\zeta_{1}-\zeta_{0}\right)^{2} / \zeta$ is so small as ordinarily to be negligible in nearly;all practical cases, since if the expenditure were double and triple we shall have only the following percentage of error in (16), viz.,

$$
\zeta_{0}: \zeta_{1}=1: 2 ; \text { error }=5.72 \% ; \text { if }=1: 3 ; \text { error }=13.40 \%
$$

It is evident that, since in formula (11) this error of weight enters into both the numerator and denommator, its effect must be greatly roduced, and it will lead only to a very small relative error indeed. 'In other words, in (11) we may always take

$$
\text { (17) } \ldots . u=\frac{1}{2}\left(\dot{u}_{0}+u_{1}\right) ; v=\frac{1}{2}\left(v_{0}+v_{1}\right) ; \text { etc. }
$$

Thus in the expression for the logarithm of the index-number, viz.,

$$
\left(\sqrt{\zeta_{0} \xi_{1}} \log . p+\sqrt{\eta_{0} \eta_{1}} \log . q+\text { etc. }\right) /\left(\sqrt{\zeta_{0} \zeta_{1}}+\sqrt{\eta_{0} \eta_{1}}+\text { etc. }\right)
$$

no considerable error will be introduced by using arithmetic instead of geometric means, and the computation will be simpler. In order to illustrate this, we may revert to the former example, and consider two commortities whose price-ratios are, as before, 1.2 and 1.6 at the end of some period as compared with the beginning. Let us further take the extreme case where the expenditure on the first commodity is trebled, and that on the second commodity doubled. since this will severely test the validity of the essumption. Thus $\zeta_{0}=50 ; \zeta_{1}=150 ; \eta_{0}=15 ; \eta_{1}=30$; $\log . p=0.07918 ; \log \cdot q=0.20412 ;$ then the two values for the logarithm of the index-number become :-

For log. of geometric mean:-

$$
\frac{50 \sqrt{3} \log .1 .2+15 \sqrt{2}-\log .1 .6}{50 \sqrt{3}+15 \sqrt{2}}=\frac{86.602 \log .1 .2+21.213 \log .1 .6}{107.815}
$$

For $\log$ of arithmetie mean :-

$$
\frac{100 \log .1 .2+22.5 \cdot \log .1 .6}{122.5}=0.10213
$$

These logarithms correspond to index-numbers, which multiplied by 100 , as is usual, are 126.99 and 126.51 respectively, the two results being therefore sensibly identical.
7. Frror of Arithmetic Means.-It is worth while to investigate, on the lines of the last example, the amount of error introduced into the logarthm of the priceindex by taking arithmetic instead of geometric means of expenditure.

Suppose, as before, there are two commodities whose price-ratios at date 1 are $p$ and $q$ as compared with unity at date 0 . Suppose that the oxpendituros at date I are respectively $k^{2}$ and $l^{2}$ times expenditure at date 0 .

By taking arithmetic means the logarithm of the price-index at date I becomes :-

$$
\log J=\frac{\left(1+k^{2}\right) \log p+\left(1+l^{2}\right) \log q}{\left(1+k^{2}\right)+\left(1+l^{2}\right)}
$$

By taking geometric means, the logarithm of the price-index at date 1 becomes :-

$$
\log J^{\prime \prime}=\frac{k \log p+l \log . q}{k+l}
$$

If $B$ denote the error introduced by takang arithmetic means,

$$
\begin{align*}
& \cdots p=\frac{k \log \cdot p+l}{k+l} \log q-\frac{\left(1+k^{2}\right) \log \cdot p+\left(1+l^{2}\right) \log \cdot q}{\left(1+k^{2}\right)+\left(1+l^{2}\right)}  \tag{18}\\
& =\{\log \cdot q-\log \cdot p) \cdot \frac{k-l}{k+l} \cdot \frac{k l-1}{k^{2}+l^{2}+2} .
\end{align*}
$$

Now we have the mequality

$$
\left(l^{2}+k^{2}\right)>2 k l ; \text { consequently }\left(k^{2}+l^{2}+2\right)>2 k l+2
$$

and therefore

$$
\text { (19) } \ldots, \ldots, E<\frac{(\log . q-\log . p)}{2} \cdot \frac{k-l}{k+l} \cdot \frac{k l-1}{k l+1}
$$

This presupposes, of course, that the mitial expenditure on each commodity at date 0 was umty.

If the initial expenditures on each commodity, instead of being unity, were respectively $c$ and $f$, then the expenditures at date 1 would be ek and $f l^{2}$. In this case we have

$$
\begin{equation*}
\ldots+E=(\log \cdot q-\log \cdot p) \cdot \frac{e f(h-l)(k l-1)}{(e k+f l)\left(e k^{2}+f l^{2}+e+f\right)} \tag{20}
\end{equation*}
$$

In this case the inequality becomes $\left(e k^{2}+f l^{2}\right)>2 \sqrt{e f} . k l$; and $(\varepsilon+f)>2 \sqrt{e f}$;

$$
\text { consequently }\left(e k^{2}+f^{2}+e+f\right)>2 \sqrt{e f}(k l+1)
$$

Also it can be shewn algebraically that if $\left(e k^{2}-f l^{2}\right)$ and ( $e-f$ ) are of the same sign, as is most frequently the case, then

$$
(e k+f l)>\cdot \sqrt{e f}(k+l)
$$

for $\left(e k^{2}-f l^{2}\right)(e-f)>0$; consequently $\left(e^{2} k^{2}+f^{2} l^{2}\right)>e f\left(l^{2}+k^{2}\right)$ and $\quad\left(e^{2} k^{2}+f^{2} l^{2}+2 e f k l\right)>e f\left(l^{2}+l^{2}+2 k l\right) ;$ and therefore

$$
(e k+f l)>\sqrt{e f}(k+l)
$$

From this analysis it is evident on reverting to (20) that

$$
E<\frac{\log . q-\log . p}{2} \cdot \frac{k-l}{k+l} \cdot \frac{k l-1}{k l+1}
$$

as in the former case; see (19).
A superior limit has thus been found for the error in the logarithm of the priceindex. As in practical examples $k$ and $l$ are 'ordmarily nearly equal, the error is thus very small, since $k \rightarrow l$ will then nearly vanish. A considerable list, viz., of about 50 commodities shews that the error is by no means inconsiderable even when the number of commodities is large.
8. Index-Numbers referred to Average Conditions during a Period. We have already shewn that the best weight to be adopted in deducing the price-indexes of any two dates is in proportion to the mean of the expenditures, and that no sensible error is involved in taking the arithinetic mean, if the computation as between the priceratios be made on the principle of the geometric mean. But the comparison of the highest value is clearly that based on the average expenditure of a longer period, smee the variations of this are less marked, being free from what may be called " large accidental departures from the mean.". Hence it is preferable to employ a qutinquenmum or decennium as basse period. And since it has been established that, for a period covering two dates, the exact nature of the determination of the mean is the weighting to be adopted (i.c., whether geometric, arithmetic, etc.) is not of high moportance, we may get results of a very high order of accuracy in a simple mannor. Thus although a strict adherence to theory demands that the logarithms of the price-ratios should be weighted by the geometric means of the two expenditures, still a result identical for all practical purposes can be obtanned by using the arithmetic means, and because of the considerable saving in computation secured by using the arithmetic mean of the weights, it is to be preferred. By similar reasoning, the proposition established can be extended to meet the case of a large number of years, instead of only two, and the conclusion is thus reached that if $s_{0}, \zeta_{2}, \ldots \zeta_{m-1}$, are the expenditures at $m$ observed periods, the general weighting may be found by taking the arithmetic mean ${ }_{m}^{1}\left\{\zeta_{0}+\zeta_{1}+\ldots . \zeta_{m-1}\right\}$, instead of the theoretically-more-accurate geometric mean $\left(\zeta_{0} \zeta_{1} \ldots \zeta_{m-1}\right) \frac{1}{m}$. This is really equivalent to asserting that the basis of the comparison of the purchasingpower of money may be the arithmetic average of the expenditure on the various commodities throughout the perod under examination.
9. Differences between various Price-Indexes.-Price-indexes may be said, in general, to purport to represent the relative amount of money that must on the average be paid for commodities at successive dates, the value paid on the original date being taken as 100. Price-ratios are similar to the index-numbers, or priceindexes, but apply only to indivadual commodities or limited groups of commodities. Since the purchasing power of gold in regard to a particular commodity is an individual measure of its exchange-value (i.e.. of the exchange-relation, between the two) it has bean commonly imagined that by taking a sufficient number of commodities a general measure of all changes in the purchasing-power of gold can be

## Appendix.

determined. In other words, it is supposed that the price-indexes represent the quantity of gold corresponding to 100 units thereof ( $£$ ) at the initial date, viz., that corresponding to the 100. An examination of the various tables of price-indexes shews that attempts to measire this general relation are very unsatisfactory. To illustrate this the tables hereunder are given. They furnish the price-indexes established by various authorities by computation from various series of commodities, and it is indicated in the tables on what the estimate is based. It will be seen that there are marked divergences between individual results, so great indeed as to indicate that therr value is very limited. For one example one series of indexes represents rises, while for the same period another will represent falls. The fact is this, viz., that price-indexes are definite only for a definite regimen, that is for a series of commodities used in given quantitues; and the hope to obtain a general price-index which will represent in its totality the variation in the general exchangevalue of gold is to expect the impossible.

No doubt for each country a series of commoditues and system of weights might be taken as representing the average usage of the entire population in regard to these commodities. Other series of commodities and systems of weights would represent the usage of the different classes in the communty. Both would differ as between nation and nation; consequently if any international standard is to be developed for the widest system of comparisons, the series should be common to all, and the weights should represent the average usage of the nations included, For international comparisons of classes a similar standard-sernes and average-weights would berequired. This has been dealt with elsewhere by me. It will suffice here to observe that a system rendering general international comparisons possible, and also international comparison of classes, would have to be estabhshed by an international practice. This could be reached only by an international commission on the subject.

The following tables give the price-indexes furnished by various authorities. They disclose the fact that they are of little value to determine quantitatively small differences of the purchasing efficiency of money, the fact being that such indexes are not sufficiently well-determined to answer many social-economic questions that are arising, for example, an automatic variation of wage-determinations, wheh has been suggested in this country (Australia). The tables enable one to obtain an idea also of the range of uncertainty as among the methods adopted by difforent authorities.

TABLE I.-VARIOUS PRICE INDEXES, 1900.1910. REDUCED TO 1900 VALUES AS BASIS.

| Year. | A. | B. | C. | D. | E. | F. | G. | 230 Com . |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 22 Com. | 45 Com. |  | 37 Com . |  |  | 257 Com . | H. <br> Weighted.* | Un. weight'd. |
| 1900 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 1901 | 99.5 | 96.9 | 101.9 | 93.5 | 96.9 | 95.2 | 98.2 | 100.2 | 98.9 |
| 1902 | 91.3 | 96.5 | 101.6 | 92.6 | 95.8 | 90.8 | 102.2 | 103.6 | 100.7 . |
| 1903 | 93.3 | 96.9 | 103.2 | 92.6 | 94.2 | 90.7 | 102.8 | 103.7 | 102.1 |
| 1904 | 102.6 | 98.3 | 104.3 | 93.6 | 97.8 | 91.8 | 102.3 | 104.5 | 103.0 |
| 1905 | 99.5 | 97.6 | 103.7 | 96.5 | 98.5 | 91.7 | 104.9 | 107.6 | 105.3 |
| 1906 | 108.7 | 100.5 | 103.2 | 102.7 | 103.3 | 97.0 | 110.9 | 113.5 | 110.9 |
| 1907 | 116.9 | 105.7 | 105.8 | 106.2 | 107.7 | 101.9 | 116.3 | 122.1 | 116.6 |
| 1908 | 107.7 | 102.8 | 108.4 | 97.5 | 103.3 | 97.9 | 111.1 | 118.2 | 111.6 |
| 1909 | 102.6 |  | 108.2 | 99.0 | 105.6 | 94.0 | 114.5 | 119.4 | 112.0 |
| 1910 | 111.8 | - | 109.9 | 103.8 | - | -. | 119.1 | - | - |

* Weighted according to table of the British Absociation for the Advancement of Seience, 1887 to 1890
A. The Economist (Old Basisi; Wholesale Prices Index Number, 1st January of
each Xear; 22 Commodities.
B. Board of Trade. Wholesale Prices in United Kingdom; 45 Commodities.
C. Board of Trade. Petail Prices in London.
D. Sauerbeck. Average Prices in England. 37 Commodities.
E. United Kingdom. From Parliamentary Paper Cd. 4867. Imports.
F. United Kingdom. From Parliamentary Paper Cd. 4867. Exports.
G. United Statest Wholesale Prices; 257 Commodities.
H. Canada. Wholesale Prices; $230^{4}$ Commoditıes. Weighted.
I. Canada. Wholesale Prices ; 230 Commodities. Unweighted.

TABLE II.-VARIOUS PRICE INDEXES, 1871-80.
REDUCED TO 1871 AS BASIS.

| Year. |  | $\begin{gathered} \mathrm{N} . \\ 45 \\ \text { Com. } \end{gathered}$ | $\begin{gathered} 0 . \\ 39 \\ \text { Com. } \end{gathered}$ |  | $\begin{gathered} \text { Q. } \\ 114 \end{gathered}$ Com. | $\begin{gathered} \mathbf{R} . \\ 50 \\ \text { Com. } \end{gathered}$ | $\begin{gathered} \mathrm{S} \\ 223 \\ \mathrm{Com} . \end{gathered}$ | T. 223 Com. | $\begin{gathered} \text { U. } \\ 223 \\ \text { Com. } \end{gathered}$ | $\begin{gathered} \mathrm{V} \\ 223 \end{gathered}$ Com. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1871 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 1872 | 109.5 | 107.2 | 109 | 106.6 | 106.8 | 105.4 | 102.1 | 97.9 | 97.3 | 103.5 |
| 1873 | 113.8 | 112.3 | 111 | 106.1 | 108.9 | 110 | 101.1 | 96.$]$ | 94.8 | 99.3 |
| 1874 | 111.6 | 108.9 | 102 | 97.4 | 107.2 | 104.5 | 97.8 | 96.6 | 95.5 | 97.1 |
| 1875 | 107.3 | 103.4 | 96 | 98.2 | 102.2 | 99.1 | 93.8 | 96.0 | 94.7 | 92.1 |
| 1876 | 104.7 | 101.5 | 95 | 96.2 | 101.0 | 92.7 | 86.9 | 92.6 | 90.1 | 95.3 |
| 1877 | 104.8 | 104.1 | 94 | 98.2 | 100.5 | 93.6 | 81.5 | 87.7 | 83.5 | 84.9 |
| 1878 | 98.6 | 97.5 | 87 | 85.4 | 94.9 | 87.3 | 74.5 | 82.6 | 76.9 | 81.3 |
| 1879 | 85.9 | 93.1 | 83 | 83.1 | 92.2 | 83.6 | 71.0 | 77.4 | 69.8 | - |
| 1880 | 99.5 | 95.3 | 88 | 85.0 | 96.0 | 87.3 | 78.6 | 82.9 | 77.1 | - |

M. The Economist (Old Basis): Wholesale Prices; 22 Commodities.
N. Board of Trade. Wholesale Prices un United Kingdom ; 45 Commodities.
O. Sauerbeck. Average Prices in England; 39 Commodities.
P. Palgrave's French Prices; 22 Commodities.
Q. Soetbeer's Hamburg Prices; 114 Cormmodities.
R. Mulhall. "Ratio of Values"; 50 Commodities.
S. Aldrıch Report ; 223 Commodities-Commodities Unweighted.
T. " " " Commodities Weighted according to Uniform Expenditure.
Commodities Weighted according to 68.6\% of Total Expenditure.
V. " ", " Gold Index Numbers. All commodities averaged simply.
Reverting to Table $I$, it is obvious that weighting is not a matter of indifference even with a large number of commodities : see coltumns $H$ and $I$, years 1900 and 1901, for example. Further, it is evident that the effect of ignoring weighting may be relatively large: see for example the year 1909 in the same columns, giving 119.4 for the weighted, and 112.0 for the unweighted results. It is clear from the same table (see columns A, B, C, D, for example, year 1904) that the indications of tables as now prepared are of relatively small value for deducing reliable estmates.

A comparison of the results on Table II. leads to the same conciusion, viz., that the divergences between different estmmates of a price-index are so great as to indicate that at present they are of very limited value.

It will be appropriate to indicate the nature and defects of various methods of computing a price-index. In this connection it may be remarked that if I, J, K, etc., are price-indexes for any series of dates, then the scheme of computation should bo such that the ratios $\mathbf{I} / J, I / K, J / K$, shall remain unchanged in whatever order the results are obtained.

Some remarks are added later concerning a supposed demonstration that the geometric mean is unsatisfactory.
10. Various Methods adopted for measuring the Exchange-Value of Money.The following are various methods which have been employed for determining the variations in the exchange-value of money. The essential features of each method , are given un terms of the notation employed hereinbefore. The notation used is that of $\S 2$, and the products $a a, \beta b$, etc., therefore measure the money-valueimportance of the different commodities at times shewn by the suffix employed. They are denoted by $\zeta_{0}, \eta_{0}$, etc., $\zeta_{1}, \eta_{1}$, etc., according to the dates. See formula (13) and ( $13 a$ ).
(i.) Dutot's Method.-In this method the prices of commodities are taken at their market quotations, and the mass-units are assumed to be equal. Then if $P_{1}$ and $P_{0}$ are the price-indexes at dates 1 and 0 ,

$$
\begin{equation*}
\frac{P_{1}}{P_{0}}=\frac{a_{1}+b_{1}+\text { etc. }}{a_{0}+b_{0}+\text { etc. }}=\frac{a_{0 * 0} p_{1}+b_{0.0} q_{1}+\text { etc. }}{a_{0}+b_{0}+\text { etc. }} \tag{21}
\end{equation*}
$$

This method consequently weights the price-ratios with the numbers $a_{0}$, $b_{0}$, etc., viz., the prices at date 0 . The method is probably now rarely used.

## Appendix.

(ii.) Carli's Method.-This method consists simply in taking the arithmetic mean of the different price-ratios and is expressed algebraically as follows:-

$$
\begin{equation*}
\frac{P_{\mathrm{I}}}{P_{0}}=\frac{\mathbf{1}}{n}\left(\frac{a_{1}}{a_{0}}+\frac{b_{1}}{b_{0}}+\text { etc. }\right)=\frac{1}{n}\left({ }_{0} p_{1}+{ }_{0} q_{1}+\text { etc. }\right) \tag{22}
\end{equation*}
$$

(iii.), Evelyn's Variation of Carli's Method.-In Evelyn's variation several periods are compared with the first, all the prices of which are taken as 100. Suppose that we have

$$
\frac{P}{P_{0}}=\frac{1}{n}\left(\frac{a_{1}}{a_{0}}+\frac{b_{1}}{b_{0}}+\text { etc. }\right) ; \quad \frac{P_{2}}{P_{0}}=\frac{1}{n}\left(\frac{a_{2}}{a_{0}}+\frac{b_{2}}{b_{0}}+\text { etc. }\right) ;
$$

then it follows that--

$$
\begin{equation*}
\frac{P_{2}}{P_{1}}=\frac{\frac{a_{2}}{a_{0}}+\frac{b_{2}}{b_{0}}+\text { etc. }}{\frac{a_{1}}{a_{0}}+\frac{b_{1}}{b_{0}}+\text { etc. }}=\frac{\frac{a_{1}}{a_{0}} \cdot \frac{a_{2}}{a_{1}}+\frac{b_{1}}{b_{0}} \cdot \frac{b_{2}}{b_{1}}+\text { otc. }}{\frac{a_{1}}{a_{0}}+\frac{b_{1}}{b_{0}}+\text { otc. }} \tag{23}
\end{equation*}
$$

Consequently the expressions $\frac{a_{2}}{a_{1}}, \frac{b_{2}}{b_{1}}$ instead of being weighted evenly (the essential feature of Carli's method) are weighted according to the numbers $a_{1} / a_{0}$, $b_{1} / b_{0}$, etc., i.e., according to the price.ratios between 1 and 0 . This points to an inconsistency in Carl's method, which is repeated in Young's method, to which reference will now be made.

1
(iv.) Young's Method.-In this method prices at the first period are taken as unity, and at the second period as $a_{1}^{\prime}, b_{1}^{\prime}$, ete. These last values are weighted according to the relative total-exchange-values of the classes in general use (at some poriod), and the sum of the products divided by the sum of the weights. Algelbraicaily it is expressed thus :-
(24)

$$
\frac{P_{1}}{P_{0}}=\frac{\zeta a_{1}^{\prime}+\eta b_{1}^{\prime}+\text { etc. }}{\zeta+\eta+\text { ete. }}
$$

$a^{\prime}$, denoting the value of $\frac{a_{f}}{a_{0}}$ when $a_{c}$ is taken as unity,
Young's methorl, however, has tho same inconsistency as Carli's. for

$$
\frac{P_{2}}{P_{0}}=\frac{\zeta a_{2}^{\prime}+\eta b^{\prime}+\text { etc. }}{\zeta+\frac{b^{\prime}}{\eta}+\text { etc. }} \text { consequently }
$$

$$
\begin{equation*}
\frac{P_{2}}{P_{1}}=\frac{\zeta a^{\prime}{ }_{2}+\eta b^{\prime}{ }_{2}+\text { etc. }}{\zeta a_{1}^{\prime}+\eta b_{1}-{ }^{2}+\text { etc. }}=\frac{\zeta_{a^{\prime}} 1 \cdot \frac{a_{2}}{a_{1}}+\eta b_{1}^{\prime} \cdot \frac{b_{2}}{b_{1}}+\text { etc. }}{v a_{1}^{\prime}+\eta b_{1}+\text { etc. }} \tag{25}
\end{equation*}
$$

In othor words, the weighting is now $\zeta a^{\prime}, \eta b_{1}^{\prime}$, instead of $\varsigma^{\prime}, \eta$, ote.
(v.) Scrope's Method. The essential feature of Scrope's method is that the same mass-units are employed at different periods. Algebraically it may be written as follows :-

$$
\begin{equation*}
\frac{P_{1}}{P_{0}}=\frac{a a_{1}+\beta b_{1}+\frac{\text { ete. }}{a a_{3}}+\beta b_{0}+}{\text { ote. }} \tag{26}
\end{equation*}
$$

that is to say, it is what has been called in $\$ 2$ herein, the "cost-of-living" formula (2). This is equivalent to the following :-

$$
\begin{equation*}
\frac{P_{1}}{P_{0}}=-\frac{a a_{0} \cdot \frac{a_{1}}{a_{0}}+\beta b_{0} \cdot \frac{b_{1}}{b_{0}}+\text { etc. }}{a a_{0}+\beta b_{0}+\text { etc. }} \tag{20a}
\end{equation*}
$$

This latter formula shows that the price-ratios are weighted by the multipliers $a a_{\beta}, \beta b_{0}$, etc., which would represent the orginal expenditures if $a, \beta$, etc., were the original quantities or mass-unts, or the average expenditures if $a, \beta$, otc., are suitably taken. Thus it resembles Young's method in form. We shail shew later that it is really the best form.
(vi.) Laspeyres' and Paasche's Variation of Scrope's Methods and Scrope's
" Emended Variation."-It may be remarked that the question of the exact massquantities to be used has not yet been touched. Threo methods are possible :-m
(a) By using the mass-quantities of the instial period-

$$
\begin{equation*}
\frac{P_{i}}{P_{0}}=\frac{a_{0} a_{1}+\beta_{0} b_{1}+\text { etc. }}{a_{0} a_{0}+\beta_{0} b_{0}+\text { etc. }} \text { (Laspeyres' variation); } \tag{26b}
\end{equation*}
$$

(b) $\mathrm{By}^{\prime}$ using mass-quantitics of the final period;

$$
\begin{equation*}
\frac{P_{1}}{P_{0}}=\frac{a_{1} a_{1}+\beta_{1} b_{1}+\text { etc. }}{a_{1} a_{0}+\beta_{1} b_{0}+\text { etc. }} \text { (Paasche's variation); } \tag{26c}
\end{equation*}
$$

(c) By using some mean between the two. The best known is the geometric mean, viz.,

$$
(26 d) \ldots \ldots \cdot \frac{P_{1}}{P_{0}}=\frac{\sqrt{a_{0} a_{1}} \cdot a_{1}+\sqrt{\beta_{0} \bar{\beta}_{1}} \cdot b_{1}+\text { etc. }}{\sqrt{\pi_{0} a_{1}} \cdot a_{0}+\sqrt{\beta_{0} \beta_{1}} b_{0}+\text { etc. }} .
$$

which is known as Serope's "emended variation," see formula (13) (13a) and (14) hercinbefore, where it has already heen shewn that tho more convenient arithmetic mean of $a_{0}$ and $a_{1}$, ote. is sufficiently accurate.
(vii.) Drobisch's Method.-.This method is the best known example of the mothods depending on double-weighting. Drobisch took his prices to be prices of thic samo aforcgated mase-unat, that is a unit consisting of all the commodities in the relative quantities as used. His method assumes that the average price of an aggregated inass-unit will be as follows, viz. :-

$$
\begin{aligned}
& \frac{a_{0} a_{0}+\beta_{0} b_{0}+\text { etc. }}{a_{0}+\beta_{0}+\text { ete. }} \text {, at the first period; and } \\
& \frac{a_{1} a_{1}+\beta_{1} b_{1}+\text { ete. }}{a_{1}+\beta_{1}+\text { cte. }} \text {, at the second period; }
\end{aligned}
$$

and so on. From this we have directly

$$
\begin{equation*}
\frac{P_{1}}{P_{0}} \frac{\frac{a_{1} a_{4}+\beta_{1} b_{1}+\text { etc. }}{a_{1}+\beta_{1}+\text { etc. }}}{\frac{a_{0} a_{0}+\beta_{0} b_{0}+\text { etc. }}{a_{0}+\beta_{0}+\text { etc. }}} \tag{27}
\end{equation*}
$$

This equatson is equal to-

$$
\begin{equation*}
\frac{P_{1}}{P_{0}} \frac{\dot{a}_{1} a_{1}+\beta_{1} b_{1}+\text { etc. }}{a_{0} a_{0}+\beta_{0} b_{0}+\text { etc. }} \cdot \frac{a_{0}+\beta_{0}+\text { etc. }}{a_{1}+\beta_{1}+\text { etc. }} \tag{27a}
\end{equation*}
$$

It is olsviously a fallacy to suppose that differently constituted "aggregated-mass-unts" can bo compared in thus way : see remarks in the next sub-section.
(viii.) Lehr's Method.-Lehr's method, as Drobisch's, also employs doubleweighting, but differs from Drobisch in the second factor on the right hand side of (27a). Algebraically it may be written-

$$
\begin{equation*}
\ldots \stackrel{P}{1}^{P_{0}}=\frac{a_{1} a_{t}+, \beta b_{1}+\text { etc. }}{a_{0} a_{0}+\beta_{0} b_{0}+\text { etc. }}=\frac{a_{0}\left(\frac{a_{0} a_{0}+a_{1} a_{1}}{a_{0}+a_{1}}\right)+\beta_{0}\left(\frac{\beta_{0} b_{0}+\beta_{1} b_{1}}{\beta_{0}+\beta_{1}}\right)+\text { etc. }}{a_{1}\left(\frac{a_{0} a_{0}+a_{1}}{a_{0}+a_{1}}\right)+\beta_{0}\left(\frac{\beta_{0} b_{0}+\beta_{1} b_{1}}{\beta_{0}+\beta_{1}}\right)+\text { etc. }} \tag{28}
\end{equation*}
$$

Lehr's method uses the arithmetic average, firstly with double-weighting, secondly on the mass-units that have the same average price over both the periods compared. It is also unsatisfactory, the objection to the methods of both Drobisch and Lohr being that were an equality of prices at two periods accompanied by a large differonce (increase) in mass-quantities, it would lead to a difference.(increase) in the price-index. That is to say, though the price of every commodity might remain the same, the formulae both of Drobisch and Lelr would furnish different price-indexes.
11. Broneously Alleged Defects in the Geometric Mean.-Laspeyres (a profeasor in the University of Basle) urged that the geometric mean, suggested by Jevons, was defective, supporting his contention by the following argument :-He supposes that from date 0 to date 1 the price of commodity $A$ advanced from 1 to 2 ,
and commodity B declined from 1 to $\frac{1}{2}$. Smee to purchase a unit of each commodity, 2 money-units would have been required mitially, and at the second dute $2 \frac{1}{2}$ moneyunits, he argues that the prices have advanced from 2 to $2 \frac{1}{2}$, that is $25 \%$. Thas, of course, is what is given by formula (2) heren, which ltmuts the consuderatron to the case where the mass unats purchased are constantly the same. In this case there can be no doubt as to which is the correct formula, in other words, the second aggregate of expenditure over the first aggregate is the only correct mode of computing the ratio of advance. But if, on the other hand, the general case is to be considered, where the degree of usage of each commodity may possibly have changed betwoen the two periods. we cannot then assume that the mass-anits are to be regarded as equal. The werghts for price-ratios are expenditures, and in the case supposed by Laspeyres they are not equal. In this mstance the " wetghts" at date 0 are the same for commodity A and $\mathbf{B}$, but at date 2 the "weights." have materially changool. If we take the "weighting" into account, then the geometric mean of the weights will give results very approximate to those which Laspeyres claims should bo given, and yet the case is not quite so limited as his was. The ittustration confirms the view that in the gencral case, the geometric mean gaves undoubtedly the better result, and Laspeyres' case does not really dispose of Jevons' argument : all it shows is that when price-ratios are used, proper weighting is no less important than m any other case, contrary to popular economic opinion. Thus by formula (2) we have the price-mdex 125 (Laspeyres' alleged correct value). But using geometric mean weights we get-

| Commodity. | Date 0. | Date 1. | Price Ratio. |  | Weights. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 1 | 2 | ${ }_{0} p_{1}=2$ | $u_{0}=1 ; u_{5}=2$ |  |
| $\mathbf{B}$ | 1 | $\frac{1}{2}$ |  | $0 g_{1}=\frac{1}{2}$ | $v_{0}=1 ; v_{1}=\frac{1}{2}$ |

Hence the result by the geometric mean, with geometric mean weights $=125.99$.
We see from this that Laspeyres' argunent fails wholly, if ns was originally pointed out, it is remembered that eomparssons are invalid which take no account of those variations in the relative importance of commodities, which may be desoribed as changes in the standard of expendrture, similar for example to changes in the standard of living or regimen. In other words, Laspeyres' contention that the geometric mean by Jevons' method gives no change of price-2ndex, is merely a consequence of an mappropriate method of deducing a price-index, and confirms the view hereinbefore expressed, that exact "woights" must be used, if the deduced price-indexes are to have any economic value. Jevons' own suggestion, that perhaps the harmonce mean may be taken, is in general irvalid, for reasons already molicatod.
12. The Aggregate-Expenditure Method, the Best.-In § 10, Scrope's mothod, Laspeyres' and Paasche's variations of this method, and Sorope's own "emended variation" have already been referred to. Scrope used the same mass-anits at 'different periods, t.e., he assumed a constont regimen. Laspeyres' variation, also based upon a constant regimen. was that he used the mass-unts of the initial period, while Paasche used those of the final period. A goometric mean between the two (even an arithmetic mean is sufficient) is perhaps more accurate. Theso four formulae are all surnmed up by formula (2), Laspeyres using as mass units $\alpha_{o}, \beta_{o}$, etc.; Paasche $a_{m}, \beta_{m}$, etc.; the geometric and turithmetic means are. $\sqrt{ }\left(a_{0} a_{m}\right)$, etc., and $\frac{1}{2}\left(\beta_{0}+\beta_{m}\right)$, etc. They are satisfactory only for any two years to be compared, but the fundamental idea for perfectly tuequivocal comparison for a series of years is the cost of a definite regimen for those years. Hence with tho aid of statistics we must make such attempt as is possible to define a regumen that may be regarded as applicable to each of the years which are to be included in a comparison.* That is, we assume mean values for a, $\beta, \gamma$, etc. Thus we use formula (2) or (26) with these mean values for the mass units.

We shall cirat shes the substantal identity of the only reasomably accurate prise-ratio method, formula (11), with the aggregate-expenditure method, formula (2). Denoting the average values of the mass+anits by a, $\beta$, etc., wo muy show $\dagger$ that if $J$ denote the result by (1]) and $I$ by (2), then

$$
\begin{aligned}
& \log J=2\left\{\frac{\alpha a x+\beta b y+\ldots}{a a+\beta b+\cdots}+\frac{\alpha a x^{3}+\beta b y^{3}+\cdots}{\alpha a+\beta b+\cdots}\left(\frac{\alpha a x^{5}+\beta b y^{5}+\ldots}{\alpha c+\beta b+\cdots}\right)+\text { etc. }\right\} \\
& \log I=2\left\{\frac{\alpha a x+\beta b y+\ldots}{a a+\dot{\beta} b+\ldots}+\frac{1}{}\left(\frac{a \alpha x+\beta b y+\ldots}{\alpha \dot{x}+\beta b+\ldots}\right)^{*}+\frac{1}{b}\left(\frac{\alpha a x+\beta b y+\ldots}{a a+\beta b+\ldots}\right)^{\delta}+\text { etc. }\right\}
\end{aligned}
$$

in which $\frac{a_{1}}{a_{0}}=\frac{1+x}{1-x}: \frac{b_{1}}{b_{0}}=\frac{1+y}{1-y}$; etc.

[^16]In all ordmary cases $x, y$, etc., are small quantities. If we suppose them equal, the two expressions are identreal. If they range in value from 1 to 10 , and $\xi \mathfrak{\xi}$ be their anean value, then the difference $\log J-\log I=-3 \frac{n-1}{n+1} \xi^{3}$ where $n$ is the number of commodities.

If we also suppose the average expenditures on the conmodities to range from 1 to 10 , then $\log J-\log I=$ about $0.56 \xi^{3}$ when $n=100$, and is never large. Rememberng that in practical examples $\xi$ can never be say $\frac{1}{5}$, in which case $\xi^{3}$ is $\mathrm{I}^{2}$, it is easy to see that the two expressions are sensibly identical for any large number of commodities.

Since the price-1ndex found from proce-ratios, using the properly weighted gemmetric mean, as sensibly identical with the price-index found from the aggregate of expenditures, ut is evdent that results by unweighted means of priceratios should be rejerted, and further that the wetghts of price-ratios are very important.

The advantages of the inethod of aggregate expendatures, formula (2), may be stated as follows :-
(i.) It is incomparably superior to the unweighted price-ratio method if the mass-units are at all near the true usage-quantities.
(it.) If the mass-umts are only approximately correct, small difierences in ther value will not sensibly vary the result.
(iin.) One can instantly see in practical computation the influence of each term on the result, abd thus estimate the effect of any uncertaintres.
(Iv.) It is the simplest possible of all methods, the precision of which entitles then to consideration.

Fimally in this connection it may be said that, reverting to formula (26a) in $\$ 10$, the " weights" or expenditures $\alpha a_{0}, \beta b_{0}$, can be made an average (or a probable averige if wo must estmate the future) and dividing these averages by $a_{0}$, $b_{0}$, we get mass-76nts, wheh must on the whole be satisfactory, and further by assuning even an approximately true regitnen. far more exact results will be obtaned than are furnished by an inperfectly weighted price-ratio combination.
13. Conclusion.-The conclusion of the whole matter may be stated as folluws :-
(1.) The only accurate comparison that can be made betwaen the purchasing power of gold at any two dates as one made on the basis of a definite series of commodities. The differences between different price-indexes shew that even an extensive series of commodities does not give a definite general resull.
(ii.) For international purposes it is desirable that a standard-series of commodities should be adopted, and that this standard series should be used as a basis for all international comparisons.
(iii.) That in connection with this series a definte method as to obtaining prices should be adopted so that the results in one country wall be immediately comparable with the results obtained in any other.
(iv.) The prices of individual elements in thas standard series should be weighted according to the mean usage of the whole of the countries included.
(v.) The weighted aggregate-price expressed in terms of some gold-untt (say £1) should be furnished, and the comparisons based upon the ratio of the weighted aggrogate prices, that is, according to formula (2) herein. Such a comparison 18 perfectly accurate and definite, and there is no mathematical objection to the mode of computing it.
(vi.) [n all cases where price-ratios are used, each commodity in the tabular lists should have associated with it the weight-number used in the computation of the price-index, and these numbers should be in the ratios of the expenditures on the commodities. In comparing the price-index of one period with another these changes of weights should be taken strictly into account.
(vii.) Where the werghts between the two periods differ but slightly, no appreciable error will arise by taking then arithmetic instead of the geometric means.
(viii.) Where the werghts are very different, the geometric mean should be employed. The general aggregate should be computed on the principle of the geometric mean, a.e., the logarithms should be taken out of the price-ratios, multiplied by the mean-weight; the sum of these products, divided by the sum of the weights will be the logarithm of the result required.
(ix.) Comparisons of this character assume that tho usage of tho figgregate of the commodities is everywhere the mean adopted, and are, therefore, on this assumption very accurats, so far as the mere computation method itself as concerned.
(x.) It is easily seen that, for simplicity, the price-ratio method cannot compare with the aggregate expenditure method.
(xi.) A result obtained on the lines suggested can be regarded, howevor. only as an individual ground of comparison, viz., one of a purely international character, and its intrinsic value will depend upon the extent to which the whole serves of commodities and assigned weights may be regarded as internationally sigmficant.
(xii.) Even such a basis as this will, in the lapse of time, doubtless be subject to a progressive movement, and it may become necessary to alter periodically the list of commodities as well as to vary the weights assigned to cach.
(xiii.) To the extent this alteration takes place tho new price-indexes will not be directly comparable with the old, and a special investigation would be required to connect the two.
(xiv.) The miternational basis, moreover, will, in goneral, not be the best possible or most appropriate for the individual nations m the group.
(xv.) For national purposes it would not be difficult, however, to include other necessary items.
(xvi.) For practical convenience it is eminently desirable that the intermational group-result should bo kept intact.
(xvii.) The sintable variations of weighting and melusion of other commodities for national purposes can easily be managed through repetition of conmodities with positive or negatnve weights, and the inclusion of other commodities with appropriate weights.
(xvin.) The same remarks apply matatis mutandis in regard to the preparations of price-indexes for particular elasses in a community, for it will be roadily recognisod that the purchasing efficiency of money varies from class to class, and the idea that there is a general value can be rogarded as correct only in so far as it may be conceived to apply to "an average individual" (l'homme moyen of Quetelet).
(xix.) The international comparison-basis would furnish the nom with which the price-index of each nation could bo comparod, and both it and the national price-index would furnish noms with which the results for different elasses within the community could be comparod.
(sx.) In view of the value of a properly computed price-index, the mere trouble of taking out logarithms of priees is a negligible quantity, and even this is unnecessary for the formation of price-indexes on an international basis.
(xxi.) Finally we may say that the aggregate of expenditures on a definte regimen is the only satisfactory method that is at all convenient from the standpoint of computation.

## APPENDIX IX.

# ON THE ESTABLISHMENT OF' A BASIS FOR INTERNATIONAL COMPARISONS OF THE EXCHANGE-VALUE OF GOLD, AND VARIATIONS IN THE COST OF LIVING. 

BY G. H. KNIBBS, C.M.G., F.R.A.S., F.S.S., etc., etc.<br>Federal Statistician, Australia.

## SYNOPSES

1. Introduction.
2. On tho selection of a list of commodities.
3. On the determination of the units and weights of the commodities.
4. Price-indexes deducod from aggregate-expenditures.
5. Price-mdexes from priceratios.
6. Proof that the method of price-ratios with a certain weighting is practically identical with that of aggregate-expenditures.
7. Invalidity of arithmetic mean.
8. Comparisons of price-indexes when alterations m the list of commodities or in the units adopted have been made.
9. Price-inclexes when the number of commodities is greatly changed.
10. Effect of change of regimen.
11. Eseudo-continutu of price-indexos with progressive change of regimen.
12. Suggested lists of commodities and scheme of working.
13. Conelusion.
14. Introduction.-The financial and general relations of one nation with another are now seen to be so intimately connected, that all changes in human affairs must be discassed in their broadest aspects on an mternational basis. To do this effectually it is necessary that for all matters, subject to statistical analysis, mean-values should be established which, in vartue of their nature, may constitute norms for all comparisons, and for extensive generalisations. The standard of living, the habits, tendencies, and general character, the degree of civilisation, and the financial methods of the whole of the, western eavilised world, though divergent in detans, tend inore and more to closely approximate to each other, so as to constituto the world in soune special sense an economic unity". For thin reason economac norms occupy an important position among those which should be established. These will represent not merely the experience and characteristics of a particular nation, but of tho whole aggregate of nations of which it is but an indiviclual member; and which constitute the international solidarity, and arnong the economic norms, a series of numbers (price-indexes) which shall roveal the vamations of the exchange value of the basis of the monetary system (gold), necessarily takes the place of highest importance. Reflexions upon the whole matter disclose the fact that we have arrived at that stage of world-development when it has become necessary to enlarge Quetelet's iden of the "average man " (l'homme moyen) to include the idea of a representative man of large groups of nations; indeed we must also create the idea of the "average nation" (la nation moyenne). This "average nation," its constitution and general
characteristics, will represent the entire western world and will constitute the proper norm for the study of the deviations of individual nations or lesser commanitics forming the combmed group. It is evidently of the hughest value for all comparative studies of national characteristics.

It will often be essential, or at least desirable, to compare smaller communties, within the nation to which they belong, not only with the international norms (the characteristics of the average nation, but also with the characteristics of the nation of which they form a part. By these two processes we may arrive at the highest form of generalised statistical knowledge.

What has been stated above may be regarded as the fundamental principle to be appled in the statistical methods of the western world as it is now constituted. It aione recognises the essential solidarity of that world, and that the significance of national variations from the international average, can be duly appreciated only by comparision with such average.

In this connection it may be observed that one of the most far-reaching elements aroong the relations of nations is that which touches the phenomenn of the fhuctuations in the exchange-values of commoditues. The most general expression for this is in terms of money, viz., pruce, since money, being the medium of exchange for all commodities, has in consequence become the common measure of their value. Thus price, oxpressing inversely the exchange-value of the medium of exchange (gold) against any commodity, enables the exchange-relation between all commodities to be imnediately deduced.*

It is convenient sometimes to follow, for certan purposes, the fluctuation in its exchange-value of the gold-unit rather than to follow the couse of prices. In other cases, however, prices serve most readily for such comparisons as aro required. Again, we may combine commodities to form a group and fix our attention on the varying guantities of gold necessary to purchase such given group. This idea we shall see 13 of the highest practical importance.

For all international comparisons of exchange-value it is self-evident that there must be a common basis in respect of the commodities solected to measure the variations of that value. Unless the basis be identical for each coumtry, the results must neeessarily be dubious; that is to say, it will become impossible to cloarlyं distinguish between the extent to which difforences in the exchange-value of gold are due to differences in the aggregate of commodities. or are due to other phenomona affecting the exchange-value, for example variations in the quantity of gold available for currency purposes, changes in the velocity of the movement of carrency, or such changes as extensions or contractions of credit, etc., all of which are variations in the effective supply of the medium of exchange.

The common basis referred to, in order to be of real value, must be sufficiently extended and so weighted as regards its individual elements, as to represent the usaye of the aggregate of the nations grouped, or what is the same thing, the usage of the "average nation." It is further necessary that this one basis should be mantained for the whole period which a particular scheme of unequivocal comparisons iss designed to include.

Here, however, a difficulty arses. It is no less obvious that to maintan the roality of the comparison, the basis must change if the usage of mankind changes. A perpetually fixed basis would not represent "the usage of the average nation." It may, therefore, bo admitted that any satisfactory basis will oxhbit a slow progressive chango in regard to the elements of which it is constituted, and the weights that must be assigned to those elements. The character of this secular variation and the question as to how the exchange-value of gold 18 to be estimated when the usage of the average nation has changed, must be specially investigated. To this we shall refer later, though it will form no part of the first question for our consideration. It may, however, bo noted that since changes in the usage of the "average nation " will necessarily vary but slowly, and probably cannot be predicted with any

[^17]exactitude, results must ordinarily be elaborated on a basis lasting for a given poriod (say a decennium), that is, on a basis which will always be a little out of date. This, however, is unavoidable, and may readily be seen to constitute no serious difficulty.

The whole question thus resolves itself into the following, viz. :-
(a) The formulation of a sufficiently extensive list of commodities un common usage among the different nations included in the international group ;
(ii.) The determination of the relatave importance of these commodities either directly, or by an appropriate combination of the results for each nation, so as to ascertain the "average" importance for the whole group;
(iii.) The techrique by means of which the general result is ascertained.

It would seem that the simplest way of determining the relative importance or "weight", therefore, from an econome point of view, depends upon two elements, viz., the aggregate-asage and tho price. Thus, for example, if we divide the price of a commodity for any year by the price for some other year arbitrarily selected as a datum year, the quotient may be called the price-ratio of those years in respect of the commodity in question. Now this may be regarded as one of many possible measures (viz., through any other commodity) of the variation of the exchangevalue of the money-cuit (gold). It $1 s$ immednately obvious that the relative import. ance of a series of these measures would depend upon the relative expenditure on each coinmodity. Hence in attempting to deduce a general estimate from a series of price-ratios, we should, in order to ascertam the weight which is to bo assigned to each commodity, first have to ascertann the aggregate expenditure for the whole of the group of nations concemed, or else the average price of each commodity, and the aggregate number of unts used of each commodity.

1f, on the othor hand, we intended to base our conclusions as to variations of exchange-value on a definite average regnom of so many unts of each commodity, then wo should need etther to ascertain the number of units of each commodity in the regumen from direct statistics, or we-should have to divide tho international aggregates of expenditure by the international averages of price, to find the number of units. As already indicated, which course it is desirable should be followed, will depend upon whether the variation in the exchange-value of gold is to be evaluated from the aggregate cost of a particular regmen (i.c., of so many units of a definite serles of commodities) or is to be deduced as some "weighted " mean of a series of price-ratios.

As regards the question of relative weights, it may be remarked that there is obviously no utrinsic rolation between units, as say between a gross, of one commodity, a ton of a second, and a gallon of a third, and it is therefore evident that the only common measure of the importance 18 the money or exchange-valute of the aggregate use of the commodity. This, however, is unfortumately variable, the varistions of price themselves producmg changes of "weight." The difficulties, however, are not insuperable, for in general the "moans" for a large aggregate vary relatively slowly. We may assume therefore that it is practicable not onty to establish a list of commodities, but also to assign to the price-ratios. of each a "weight "number, expressing its importance in the enture group. It may be further noted that this series of "weight "-numbers must apply to limited periods (e.ty. for a docommun), and may then be reyised ; and it is of course possible also that the list of commodities must also be periodically revised. We can also decide on the average number of units of each used, that is, the quantities of each commodity in the averago regimen.

When a tist of commodities and the relative number of units of each used, or appropriate " weight" numbers are to hand, it is necessary then to decide upon a suitable arithnetical techmque of comparison. The only unequivecal or perfect system of obtaining comparable results is to compute the aggregate-value of the whole serios, from the number of units of each commodity corresponding to average usage, and the average price for the particular period (day, month, quarter or year, for example) which it is dessred to compare; see formula (3) $\mathrm{m} \S 3$, hereinafter. Since the arithmetical labour involved is by no means prohibitive, it may also be very desirable to watch the characteristics of monthly or quarterly fluctuations in these aggregates, for example, in order to study the variations of exchange-value of gold within the yoar itself, and the mean of the results of any smaller period would furnish the requisite mean value for a longer one. For example, the mean of the four quarterly results would give the mean value for a year.

These aggregate values deduced from the whole senes of commodities from the prices of each; and using the proper number of units of each, enable all necessary comparsons to be made with mathematical structness. We may dofino this as the " method of aggregate expenditures." This method is unquestionably better than that of using price-ratios with weights. Any year may be made a datum, and references may be made forward or backward from that year wathout in 'any way vitiating the comparison; in other words, the process in this mstance is always arithmetically consistent. Expressed as an algebraic formula, the process is us shewn hereinafter in § 3 , see (3).

A method already referred to which has been largely used and which, if properly handled, is also fairly, but never wholly, satisfactory, is to deduce tho prico-indox from price-ratios with appropriate weights. The average price of each commodity for some year is taken as a datam, and the price-rathos are ascertained by dividing the price for any other period by the price for the datum period. Tho quotient. usually multiplied by 100 , is the price-ratio of the latter date compared with former, When price-ratios are used, it may be shewn that the only proper mode of conbination is what is known as the geometric. and this method is the only one used which is arithmetically consistent. To olvtion the geometric mean each price-ratio is raised to the power indicated by the "weight," and the product of the whole of the preceratios, so raised, is a radicand of which a root, equal to tho sum of all tho weaghts, is to be taken. The indicated operation is very smply effected by taking ont the logaritisms of the price-ratios, multeplying each, by ats corresponding worght, and dividing the sum of these products by the sum of the weights.

Expressed as an algebrase formula, this last prescription is denoted by :-

$$
\text { (1) } \ldots \ldots, I=\left(p^{u} \cdot q^{v} \cdot r^{w} \text { ete. }\right) \frac{1}{u+v+w+\text { etc. }}
$$

or logarithmically-

$$
\begin{equation*}
\ldots \cdots \log I=, \frac{u \log p+v \log q+w \log r+\text { etc. }}{u+v+w+\text { etc. }} \tag{2}
\end{equation*}
$$

in which $p, q, r$, ete., denote the price-ratios of a series of commodities, $u, v, w$, ete., their "wenghts," based upon expenditures, and $T$ the price-index requirol.

This process gives values very similar to the previous one, aml is arthmetncally consistent. Attempts have been made to obtan price-indexes by multiplying each price-ratio by the corresponding " weight" and dividing by the sum of the woaghts. Such a process, however, is arithwetacally invalid, smee it gives incorrect ratios between different years. In other words, it furnishes different results according as to whether we work from the calculated general rosult or from tho original clata. This is sufficient ground for excluding the method. We shatl shew later on the nature of the arithmetical inconsistencies referred to, and it may hero be stated that the extraordinary differences in the exchange-value of gold, indicatod in the difiorent series furnished by economists, shew that some better arrangement must be made if the price-indexes or index-numbers are to have any general validity, or aro to be used critically. The most fruitful source of these differences lies in the fact that the lists of commodities are not adentical, and are subject to different weightings.

On the Selection of a List of Commodities.-It is evidont that, in order to bo comparable at any two periods, a commodity must not have matorially changed in character or quality. Certam commodaties, for example, may give less troublo in this respect than others; for examplo, such raw products as may be regarded as sensibly umform in quality, or manufactured products that do not matemally change in quality. Metallic ingots, pig iron, etc., may be takon as a far illustration of tho former, sugar, flour, etc., of the latter. But oven in regard to these, ether clifferences of quality or arbitrary preferences may cause the exchange-values to range between very wide limits.

It is well to point out here that variations of exchange-value nay by no means be wholly attributable to a'general variation in the purchasing-efficiency of gold. For example, articles in which the cost of manufacture enormously varics* will tend to reflect conspicuously every varmation in the rate of the remuneration of labour. The obyious reason of this is, that with raw materials the proportion which labour represents is usually very small as compared with what it represents in very highly elaborated products. $\dagger$

[^18]It is self-evident that, with such commodities, the governing element is the rate of remumeration for labour, and that the price of the commodity tends merely to reflect the variations of thas element.

As a consequence of the operation of influences of this kind, it would seem that in att international mquiry, enther as to wholesale or retal prices; all commodities in common usage, and of which the quality is comparable and identifiable, might probably be included, but whether this be so or not will depend upon the fundamental purpose of the inquiry.

If, however; we were compling a world-wide mdex-number, representing variations in the exchange-value of gold, it might probably be desirable not to include all commodities the qualities and characters of which are comparable and indentifiable, but merely those for which, in addition, world-markets exist. Thus it might be desirable to exclude all such vegetables, fruits, etc.; the price of which would necossarily be govemed mainly by local conditions. In a pure " cost-ofliving" comparison such commodities ard their prices could not of course be excluded.

We have seen that variations in exchange-value are not wholly attrbutable to variation $\mathrm{m}^{2}$ the purchasing-efficiency of gold against ordinary commodities, excluding labour, and further, that the object of the measure of the exchange-value varies according to the characteristics in the group of commodities by means of which it is measured.

It is clear from the considerations just indicated that the serjes of commodities should not only be individually identifiable in respect of character and quality, but should also be well selected from such point of veew as is important, otherwise the derived results will bo dubsous, and it is here that the prinempal difficulty arises, though there is no escape from it.

It must be observed at the sarne time, however, that progress in the technique of industry indicates that we can push this principle too far, a good illustration of - which would be the state ot steel manufacture before and after the introduction of the Bessemer and the Siemens-Martens processes. Other examples that might be cited are sugar, chemical products, etc.; in wheh there have been striking advances $m$ quality. The advances in techology have led in many cases to marked improvement in the quality of the manufactured articles. Since, however, the use of the article, thus improved in quality, may be continuous, and the change in quality may proceed by imperceptible changes, a feature not uncommon with regard to textiles, for example, it is not always possible to take so exact an account of differences of quality as has been indicated as nocessary.

Neglect of facts of this kind may eassly betray one moto an undue faulh an priceratios, and into the false belef that price-iatios for aggregates are unequivocally valid measuros of the variations $m$ exchange-value of gold, whereas the truth of the matter is that changes in the exchange-value of gold have been confused in the general result with variations $m$ the gualaty of the articles, and consequential changes in their cost, utility, or esteem values.

What has been said is sufficient indieation that in the selection of a series of commoditjes for the international basis, extreme care wall have to be exercised. I have suggested a series, and have incheated theur weight numbers at the end of this Appendix. This is done merely tentatively and purely by way of suggestion. It is supposed that eash item in this series $1 s$ identifialble with sufficient certanty' to make the aggregate cost of the whole seraes rehable. It cannot be too distmetly borne m mind that the difficulty is not in any way got over by the use of price-ratios, as is sometimes supposed, but is only haddcn so as to be less readily duscerned.

The question of the sigmficance of labour in the cost of commodities already referred to is worthy of special attention. We proceed to consider the matter.

The fact that cominodities differ greatly in respect of the value of the raw material of which they are composed, and the amount of labour which has to be applied' to that material in order to give them themr final form, suggests that regard should be had thereto in the scheme of classification.*

As between one commodity and another the ratio of the two varies greatly, and price will tend to reflect all variations in the remuncration of labour in proportion as the labour element in the production of the commodity is large.

[^19]From this it can be seen that it matters much whether the ain of an inquary be to ascertain the efficiency of gold in respect to commoditnes as influenced by wayes, or as not influenced by wages, and at is from this point of view that it may at once be seen how desirable it is that the hast of commodities should be so divided as to furnish series shewing progressive amounts of labour appled to their production. In this way it will become possible to detect the influence upon price of ruling rates of wages. If, therefore, a whole series of commodities be divided into several classes, each class shewing progressively larger amounts of labour, then we shall have the material for discriminating between the purchasing efficiency of gold in regard to raw inateriak and highly elaborated materials, and will have the data to ascertain how far demands for higher wages are merely equating themselves by a rise in prices. For if it were possible for the prices of commodities to rase throughout in the same ratio as wages, then there would be no advantage, the change would be merely a nominal ono. It, is from this point of view that one sees that, in so far as change of remuneration for labour results in mereased prices, the advantage tends to become unreal, and is nullified the money which is paid for labour giving to ats recipient no advantage in purchasing the commodities which satisfy his needs.

It is evident that this matter is of eminent economic importance. If m making, finally, the compartson of the price-indexes of the successive series of commodities in which the element of labour is playing a more and more conspictous part and in which consequently the influence of the remuneration of labour is more and more felt it turns out that the latter tends to closely correspond to variations in the rate of wages, then the obvious economic deduetion is that the result is due to variations in the remuneration of labour. Should the variations completely correspond with change in the cost-of-living for the class represented, the effect of rise in wages will be completely nullified by the rise in the price of the commodities used.

It will be seen from these considerations that the divisions of the list of commodities should, as far as possible, be homogeneous with respect to the relative cost of raw material to labour m the production of the commodity. We may conelnde. therefore, that so far as the selection of commodites is concerned, the following principles may serve as a guide, viz. :--
(i.) The commodities should be identifiable in respect of their essential characters.
(ii.) They should be largely used.
(iii.) The whole series should be divided roughly into groups, homogenoous with respect to the relative value of raw material, and labour applied to convert each commodity anto its final form.
(iv.) Only commoditzes which find a world-market should be used for mtermational comparisons. for variations in the exchange value of gold.
(v.) A supplementary list of commodities of local production are necessery if it be desired to determine such variations in the cost of living as may be attributable to variations in the exchange value of gold.
3. On the Determination of the Units and Weights of the Commodities.-Tho unit by means of which different, commodities are usually measured, may be volume or weught, or number of articles, etc. ; for example, in English measure, a gallon or a bushel, a pound or a ton, a gross or dozen, etc. All such quantities or units may be called mass-untts, and the number taken for each commodity should be in the ratio of their actual usage. It $1 s$ instantly evident, however, that there is no mtrinsic relationship between economic value and the mass-units of different commodities; for example, between a carat, in the measurement of the precious stonos, and a gallon m the measurement of sprits; in fact it is readity percerved that in the nature of the case there can be only one common measure for the relative economic importance of different commodities in question of varnation of ex́changevalue, and that is the product of the money-value of a unit, and the number of unts used, or upon the relative aggregate expenditure on the commodity. As prevously indicated, when we suppose the number of units, used to be constant at any two dates for which a comparison is desired, the best-in fact the only exact-comparison is the ratio of aggregate expenditure at the compared date to the aggregate expenditure at the original date. If the number of units of each commodity were not constant, then any deduced price-index would be vitiated by what may be called change of regimen. For this reason, once we decide upon the size of the unit which is to be compared, the mass-weight-number of units of usage may be detomined by dividing the total expenditure by the price, and it is to be assumed that this number of units is constantly used throughout the periods compared.

There is, however, a much simpler way of stating the whole matter, viz., the 'following :-

It is obvious that when we use price-ratios the actual size of the unit used disappears. For example, price per ton divided by price per ton 18 the same thing as prree per pound divided by price per pound. This has led to an erronoous opmion that price-ratios get rid of the necessity for considering the size of the unit, and that the wetghts assigned to the price-ratios in any computation represent the relative importance of the commodities. The relative importance, however, is measured by the aggregate expenditure smce the money-unit is the only common measure of economic value or exchange-value. If, therefore, relative aggregate expenditure on any commodity (i.e., the proportion of the expenditure on the commodity in question to the aggregate expenditure on all commodities) be equal on any two oceasions, the combming "weight" of the commodity remains unchanged, in the, computation of price-indexes from price-ratios.

From this point of view it becomes apparent that it 18 possible to compute a general variation in the exchange-value of gold with fair accuracy, allhough the regmen on successive occasions may have changed. Without doubt this fact has also given rise to the erroneous impression that price-ratios are to be preferred, and that they oscape the difficulty about a constant regimen. It may be pointed out, however, that the basis of comparison should undoubtedly be the mear-weight between the two occasions, but to take this into account the arithmetical work of companson is greatly elaborated and tends to become prohbitive. We shall return to this point later. It will suffice here to observe that a very much more convement system could be adopted, by using units of quantity which can be regarded as representing the average use of all the nations in the international comparison.

If for the aggregate of nations a list, shewing the total exponditure upon the various items of a whole series of commodities during any definite perrod of time oxisted, this would represent the usage, and furnish the required number of units, or the mass-werghts, the supposition being that that usage expressed the habit or tho necessity of the people. It would indicate the economic weight that should be attributed to the individual item, by the ratio whech expenchture on that item bore to the aggregate of all expenditures on the list. Futher, if, as is desirable, it were proferred to use uumbers of " mass-units" of each comrnodity so as to form aggregates by summing the prices multiplied by these units to form totals for the dates to be compared (the ratio giving the price-index) then all that is necossary is to divide the international aggregates of expenditures by the international average prices. The quotionts are the amts required.

It may here be observed that questions of exchange-value are very properly dissociated from those of utulity-value, esteem-value, and cost-value or other special theasures of value, for many commodities obviously have esteem-values wholly out of relation to their cost-values; n fact, business-practice endeavours to create esteen-values so markedly above cost-values as to ensure large profits to the manufacturer or supplier. In the questions with which we are dealing, however, exchangevalue is tho only valuo that neod be considered.

- 4. Price-Indexes Deduced from Aggregate-Expenditnres.-It has already been sand that much of the techmque in connection with the determination of variations in exchange-value practically involves the clouding of the real issue in generalataes; that the comparison is unreal or dubious to the extent that the regimen has changed, and that tho proference for price-ratios merely arises from the fact that the defect in tho techniquo of computing price-indexes from them has been relegated to a position where at is not discernible. In order to bring the matter into clear relief, let us take a very elemontary case where only two commodities are under consideration, and observe exactly what takes place in different methods of combination. We shall denote the basie dato by the suffix 0 attached to any quantity, and the second or later date by the suffix $i$, the two commodities being denoted by $\mathbf{A}$ and $\mathbf{B}$. We shall suppose the usage of these commodities at the two dates to be as expressed in the following schedule :-

| Commodity. | Date 0. |  |  | Date 1, |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Units. |  | Price. | Units. |  | Price. |
| A | 1 | (a) | 3 | . 1 | (a) | 6 |
| B | 2 | @ | 4 | 3 | (a) | 5 |

and let us use first the method of the ratio of aggregate expenditures which, expressed ajgebracally, is-

$$
\text { (3) } \ldots \ldots \ldots \ldots \text {. } I=\frac{\alpha a_{1}+\beta b_{1}+\gamma c_{1}+\text { ete. }}{a a_{0}+\beta b_{0}+\gamma c_{0}+\text { etc. }}
$$

in which $\alpha, \beta, \gamma$, etc., are the number of units of each commodity used at each date, and $a, b, c$, are the prices of those units, the suffixes denoting the dates. We observe first of all that in the case considered there have been changes in both regimen and price, but to determine the variation in the exchange-value of gold we must eliminate the effect of change of regimen. Let us then first consider a comparison based upon a supposed constancy of regimen. Thus we may take into account three cases, viz., where the regimen at the second date is as at tho first; where the regimen at the first date is as at the second; and where the regimen is the arithmetic mean of the regimens at the two dates. This will give us the result shewn hereunder as the effect of change of price, viz. :-
(i) the regimen consists of one unit of the first commodity, and two units of the second commodity on both dates;
(ii.) the regimen consists of one unit of the first commodity, and of three of the second on both dates; and
(iii) the regimen consists of one unit of the first commodity, and two-and-a-half of the second commodity.
We thus get the following results :-
Regimen of date 0; (i.);

$$
\begin{aligned}
& \frac{1 \times 6+2 \times 5}{1 \times 3+2 \times 4}=\frac{16}{11}=1.4545 \\
& \frac{1 \times 6+3 \times 5}{1 \times 3+3 \times 4}=\frac{21}{15}=1.4000 \\
& \frac{1 \times 6+2 \frac{1}{2} \times 5}{1 \times 3+2 \frac{1}{2} \times 4}=\frac{18 \frac{1}{2}}{13}=1.4230
\end{aligned}
$$

5. Price-Indexes from Price-Ratios.-Suppose now that we attempt to caloulate such results by means of price-ratios. We have the following price-ratios for the two dates, (1)/(0), $F$ denoting price-ratio for commodity A, and $a$ denoting priceratio of commodity $B$.

$$
\begin{array}{cccrc}
\text { Commodity A. } & \text { Price Ratio. } & \text { Commodity B. } & \text { Price Ratio. } \\
p & = & \frac{6}{3}=2 . & q, \quad= & \frac{5}{4}=1.25 .
\end{array}
$$

At the first date the aggregate expenditure was 11, of which 3 was on $A$ and 8 on $B$, therefore the relative importance for $A$ was $\frac{q^{2}}{1}$ and for $B$ 最. At the second date the aggregate expenditure was 21 , of which 6 was for $A$ and' 15 on $B$, therefore the relative importance on the second occasion was $\frac{7}{7}$ or $\frac{8}{7}$ for $A$, and $\frac{5}{5}$ for $B$. Hence the arithmetic mean of the weights was-

$$
\begin{aligned}
& \text { For A, } \frac{1}{2}\left(\frac{3}{11}+\frac{2}{7}\right)=\frac{4 .}{154}=u, \text { say ; } \\
& \text { For } B, \frac{1}{2}\left(\frac{8}{11}+\frac{5}{7}\right)=\frac{111}{154}=v, \text { say. }
\end{aligned}
$$

Hence, working by the geometrio means, we have-

$$
\log \cdot\left\{\left(p^{u} q^{v}\right)^{\frac{1}{u+v}}\right\}=\frac{\frac{43}{154} \log .2+\frac{111}{154} \log .1 .25}{\frac{43}{154^{\prime}}+\frac{111}{154}}=\frac{23.7013}{154}=0.153904
$$

$$
=\log . \text { of } 1.4253
$$

Now this last result is sensibly identical with what we found by taking the aggregate, and is nearly the mean of the results by suppositions (i.) and (ii.), viz., 1.4272. (If we take the geometric mean of the weights instead of the arithmetic mean we get :-

$$
\begin{aligned}
& \text { For } A, \sqrt{ }\left(\frac{3}{11} \cdot \frac{2}{7}\right)=0.27914 ; \\
& \text { For } B, \sqrt{ }\left(\frac{8}{11} \cdot \frac{5}{7}\right)=0.72075 .
\end{aligned}
$$

the sum of which is 0.99989 , and this gives 0.1538963 the logarithm of 1.4253 as before.)

Two things are obvious from the example furnished for the case of two commodities, viz. :-
(a) That with a large number of commodities the mean number of units used of each may be taken as a basis for computation of a price-index from the ratio of aggregate expenditures at any two dates, (formula 3); and also
(b) that price-ratios weighted in proportion to the average expenditure will yield an almost identical result.
This may be shewn formally by finding an algebraic expression for the difference ( $D$ )-
(4) . $D=\left(p^{u} \cdot q^{v} \cdot r^{w} . \text { etc. }\right)^{\frac{1}{u+v+w+\text { etc. }}}-\frac{a a_{1}+\beta b_{1}+\gamma c_{1}+\text { etc. }}{\alpha a_{0}+\beta b_{0}+\gamma c_{0}+\text { etc. }}=H-K$, say.
in which $u=\frac{1}{2} a\left(a_{0}+a_{1}\right) ; v=\frac{1}{2} \beta\left(b_{0}+b_{1}\right) ;$ and $w=\frac{1}{2} \gamma\left(c_{0}+c_{1}\right)$.

## 6. Proof that the Method of Price-Ratios, with Weighting according to Average

 Expenditure, is Practically Identical with that of Aggregate-Expenditures.--Since the method of determining price-indexes from price-ratios is commonly supposed to possess some advantages through its apparent generality, and since also such opinion is not sound, it is not unimportant to shew conclusively that it yields sensibly identrical results in all practical cases: This may be shewn formally by finding an algebraic expression for the difference ( $D$ ) above.We may put $a$ for $\frac{1}{2}\left(a_{0}+a,\right) ; b$ for $\frac{1}{2}\left(b_{0}+b_{1}\right) ;$ etc. ; and also $a_{0}=a$ $(1-x) ; a_{1}=a(1+x)$; and similarly throughont.

Then we have

$$
\frac{a_{1}}{a} / \frac{a_{0}}{a}=\frac{a_{1}}{a_{0}}=\frac{1+x}{1-x} ; \quad \cdot \frac{b_{1}}{b_{0}}=\frac{1+y}{1-y} .
$$

In all practical instances $p, q, r$, ete., do not differ greatly from unity, hence the quantities $f(1 \pm x)$, etc., can be expanded in convergent series. Thus we have to find the value of-
(4a) . $D\left\{\left(\frac{1+x}{1-x}\right) \cdot\left(\frac{1+y}{1-y}\right)^{\beta b}\right)$, etc. $\frac{1}{a a+\beta_{i} b+\text { etc. }} \frac{a a(1+x)+\beta b(1+y)+\text { ete. }}{a(1-x)+\beta b(1-y)+\text { ete. }}=H-K$, say.
The values of $\log H$ and $\log K$ are :-
(5) $\ldots \log H=2\left\{\begin{array}{l}a a x+\beta b y+\ldots \\ a a+\beta b+\ldots\end{array} \frac{1}{3}\left(\frac{\alpha a x_{+}^{3} \beta b y_{+}^{\beta}+\cdots}{a a+\beta b+\ldots}\right)+\frac{1}{5}\left(\frac{a \alpha x^{5}+\beta b y^{5}+\cdots}{a a+\beta b+\ldots}\right)+\right.$ etc.
$(5 a) \ldots \log K=2\left\{\frac{\alpha a x+\beta b y+\ldots}{a a+\beta b+\ldots}+\frac{1}{3} \cdot\left(\frac{a a x+\beta b y+\ldots}{a a+\beta b+\ldots}\right)^{3}+\frac{1}{5}\left(\frac{a a x+\beta b y+\ldots}{a a+\beta b+\ldots}\right)^{3}+\right.$ ots.
Therefore-
(6) $. \ldots \log H-\log K=\frac{2}{3}\left\{\frac{\alpha a x^{3}+\ldots \text { etc. }}{a \alpha \cdot+\ldots \text { eto. }}\left(\frac{a a x+\text { etc. }}{a a+\text { ete. }}\right)^{3}\right\}+\frac{2}{5}\{$ etc.-etc. $\}+$ ete

Thus the first and large term of the expressions for the logarithns of $H$ and $K$ agree, but the sewond and subsequent terms differ. The unequivocal condition. that these terms shall be equal is that the prices have all increased or all diminished in the same ratio, viz., that $x=y=z$, etc., in wheh case the second terms become ${ }_{3} x^{3}$ in each case, the third $x^{5}$, and so on : that is, the two expressions are then equal throughout. We shall shew also that in all practical cases they are equal; and first we note that the quantities $a a, \beta b$, etc., are always positive, but that $x, y, z$, etc., may be all positive, all negative, or may not be all of one sign : the latter is ordinarily the case. The quantities being of the same order of magnitude, it is obvious that the difference between the terms would be greatest when they

## Appendix.

are all of the same sign. We eonsider the case, therefore, where aa $=\beta b=$ etc., but $y=2 x, z=3 x$, etc., In this case the average value for $x, y$, etc., will be $\frac{1}{2}(\mathrm{n}+1) x=\xi$ say, $n$ being the number of commodities. Consequently we ehall have for the value of the two cubic terms-
$\frac{2}{3} \frac{\operatorname{aax} x^{3} n^{n} n^{3}}{a \alpha n}=\frac{1}{6 n}\{n(n+1)\}^{2} x^{3}=\frac{4}{3} \frac{n}{n+1} \xi^{3}$ and $\frac{2}{3}\left(\frac{\alpha a \Sigma_{1}^{4} n x}{a \operatorname{an} n}\right)^{3}=\frac{2}{3} \xi^{3}$
The difference, therefore, in this instance is-

$$
\log H-\log K=-\frac{2}{3} \xi^{3} \frac{n-1}{n+1}
$$

which is $3 \xi^{3}$ when $n 1 s$ infinite, and is only about $2 \%$ short of $\frac{\xi^{3}}{}$ when $n$ is 100 .
If further, we suppose that $\beta b=2 \alpha a ; \gamma c=3 a a$, etc., and $y=2 x$, 2 $\mathbf{3} x$, etc., as before, we shall have instead of the above-

$$
\frac{16}{45} \cdot \frac{2 n+11}{(n+1)^{3}}\left(3 n^{2}+3 n-1\right) \xi^{3} \text { and } \frac{16}{81}\left(\frac{2 n+1}{n+1}\right)^{2} \xi^{2}
$$

the difference of which is $\frac{1}{1}$
$\log \dot{H}-\log K=\frac{\xi^{3}}{405} \cdot \frac{2 n+1}{(n+1)^{3}} \quad\left\{144\left(3 n^{2}+3 n-1\right)-80(2 n+1)^{2}\right\}$
which has a value of about $0.56 \xi^{3}$ when $n=100$.
In examples practically, occurring we can never have the average value of $\xi$ as great as, say, $\frac{1}{4}$, viz., ats value when all commodities have on the average advanced about $50 \%$. Hence $\xi^{a}$ is less than $T_{5} \frac{1}{5}$, and in the three cases considered for 100 commodities, the difference would be 0 , or less than $3_{7}^{2} 5$ and ${ }_{2} \mathrm{t}_{5}$ respectively. This is the difference in the two logarithms, but each is ordinarily the logarithm of a number aomewhere near unity, and consequently represents approximately the ratio of the error itself.

It has thus been proved that $H$ and $K$ are sensibly equal in the circurastances of the case under consideration. It is obvious from this that, if the use of weighted price-ratios is deemed to be justified on the ground of any supposed generality in combining different measures of the exchange-value of money, then it follows from formula (6) that the method of ratios of aggregate expenditures, formula (3), is also valid. It is certainly the simpler to use, and eomputation of price-indexes is greatly facrlitatod by its use. This, however, while not unimportant, is not its chief merit, which is that the computer sees clearly what he is doing, while in the use of price-ratios it is by no means obvious that mproper weighting vitintes the resulte. It is now soen that the method of price ratios with inaccurate weights is sensibly equivalent to forming an aggregate with whits which do not represent the actual average usage.

It has been abready undicated that the weighted geometric mean of the priceratios is alone valid, and it has been shewn that the method of deducing priceindexes from the ratio of aggregates of expenditure, based on the use of a constant number of units, is substantially identical therewith. It now remains to consider the method of arithmetic means, not unfrequently used.

The following demonstration that even the weighted arithmetic mean is invalid is therefore not unmportant. That. the unweighted mean is quite invalid can be readily seen to be a consequence of the preceding proof of the approximate identity of the aggregate expendeture and price-ratio methods of deducing price-indexes. But we shall now examine the question of the legitimacy of using a werghted mean in another way.
7. Invalidity of Arithmetic Mean.-Using the suffixes preceding and following $I$, the price-index, to denote the dates to which it applies, we have by the method of the arithmetic mean of weighted price-ratios

$$
(7) \ldots \ldots{ }_{0} I_{1} \Rightarrow \frac{u p+v q+w r+\text { etc. }}{u+v} \frac{w}{+ \text { etc. }}
$$

Hence if we make date $l$ the basis, and obtain the price-index for date 0 in relation thereto, we ought to obtain by the same process-

$$
(7)+\ldots, I_{0}=\frac{u \frac{1}{p}+v \frac{1}{q}+t v \frac{1}{r}+\text { etc. }}{u+\frac{v}{1}+\frac{w}{+} \text { etc. }}
$$

since each price-ratio is the reciprocal of the former, and this expression (7a) should equal the reciprocal of the preceding one, viz., (7) that is, if the method were arithmetically valid. But if this equality held we should have-

$$
(u+v+w+\text { etc. })^{2}=(u p+w \underline{w}+w r+\text { etc. })\left(u \frac{1}{v}+v \frac{1}{q}+w \frac{1}{r}+\text { etc. }\right)
$$

We see, that so far as the sum of the squares of the quantities $u$, $v$, etc., is concerned, the two sides are identical, but so far as the products in pairs go the right hend side is always greater than the left when $p, q$, etc., are not equal. Or, limiting the consideration to two price-ratios, we have to shew that -

$$
\frac{u}{\frac{p}{u}+\frac{v}{q}} \text { shọuld equal } \frac{u+v}{u p+v q}
$$

if the method be arithmetically consistent. Maltuply both expressions by ( $u+v$ ) ( $u p+v q$ ), we then have :-
that is, $p / q+q / p$ should equal 2. It, however, always exceeds that quantity unless $p$ t $q$. The method of taking weighted arithmetic means, formula (7) is consequently arithmetically invalid, being irreversible; in other words, if priceindexes for a series of years are computed by the formula, they do not furnish the same ratios among one another as are furnished by the origmal data using the same process.

The weighted geometric mean, on the other hand, formula (1), is consistent, and furnishes a series of priee-indexes which furnish the same ratios as are furnished by the original data.
8. Comparisons of Price-Indexes when Alterations in the List of Commodities or in the Units Adopted have been made.-It has aiready been pointed out that if price-indexes are to be strictly accurate, then change of regamen, that is to say, either a change in the actual list of commodities or in the units adopted for individual members of the list, cannot be admitted, otherwise variation an the exchange-value of. gold becomes confused with the effect of change of regimen. The conception that new commodities may also furnish additional evidence of the exchange-value of gold is vald only when they belong to both periods to be compared. More definitely, if $a, \beta, \gamma, \delta$, etc., denote the numbers of units of the commodities $A, B, C, D$, etc., we cannot compare regimen $a \mathrm{~A}+\beta \mathrm{B}+\gamma \mathrm{C}$ etc., with say $\beta_{1} \mathrm{~B}+\gamma_{1} \mathrm{C}+\delta_{1} \mathrm{D}$ etc., though we could of course, as already shewn, compare regimen $\beta \mathbf{B}+\gamma \mathbf{C}$ with $\beta_{1} B+\gamma_{1} C$, the commodities $B$ and $C$ being common to both. Thus comparison can be made for example by assuming an arithmetic mean regimen, viz., $\left.\frac{1}{2} \beta+\beta_{1}\right) \mathbf{B}+\frac{1}{2}\left(\gamma+\gamma_{1}\right) \mathrm{C}+$ etc., to apply to the dates to be compared.

While the above statement is true, it is also true that the validity of any computation of price-indexes becomes of questionable value if the adopted list of commodities with assigned units of usage, (or price-ratos with their assigned weights) fails to coincide with the usage of the group of nations aggregated for international comparisons. The two things to be attended to are (i.) what may be called arithmetical validity, and (ii.) conformity to economic facts. From this point of view, it is to be regarded as inevitable that in the course of time changes will occur both in the commodities and their units of usage (or the weights assigned to their changes of price-ratios) in the international list. A revision, therefore, could perhaps be made every ten years, and the question then arises whether continuity in the ex-change-value relatron can be established, and if at all, in what way.

Let us suppose that, for one decennium, say, the comparisons have been based npon $m$ commodities, and that then a change is made, and comparisons are afterwards based on $n$ commodities. Of these $m$ and $n$ commodities let us suppose also that there are $k$ common to each series; and moreover, that the units used (or the
series of weights assigned to price-ration) used are not tho same on the two occosions. We have already shewn that m such a case we can found a comparisou only on some common regmen, preferably the arithmetic mean of the umts used (or, if priceratios are used, the mean of the weights assigned to the prices of theso $k$ commodities).

Primarnly it is to be observed that strictly we can make a comparison only through the $k$ commodities constituting that part of the regimen common to the two periods. This is evident from the fact that change of regimen produces its own effect on the aggregate of a list of commodities, or on the weighted mean of price-ratios, the exchange-value of money being constant. And it is for this reason that, if we want to compare the excharge-value at any two periods we can do so only on some givon number of units of a group of commodities existing at both periods; and to have the highest significance these assigned units of usage sibould, as near as possible, expross the actual usage at either clate, and hence may be taken as the arithmetical mean of " the units at either date or of the weights used m connection with the prico-ratios. For the method of aggregates the umbts may be the arithmetical menns of the units used in either.*

It is obvious from this that there cas be no real continuaty in a scries of price. indexes where the series of commodities used or the units of usage have changed, or where the weights assigned to the price-ratuos of individual commodities have altered. For this reason, when a change of basis is made, the results should be computed on the old basis for tho first year of the new sones. Thus for this yoar tho aggregates are formed on both bases, the one giving the elosing value of the price-molexes, and thoir valuo is the factor to be used for the results given in the now series. Tho supposition, however, that by this process the second series of price-indexes is perfectly contmbous with the old series is subject to some qualification, for the now series camot strictly be referred back in this way. A perfect comparison between any two penods can be made only on the basis of the average usage of the sernes of commoditios common to the two dates, the unte assigned being a mean of the units assigned for the two dates.

To express the wholo matter definitavely, lot $\dot{\Sigma}_{0}, \mathbf{\Sigma}_{1}$, etc., denoto respectivoly the aggregates $\alpha^{\prime} a_{0}+\beta^{\prime} b_{0}+$ etc. $; a^{\prime} \alpha_{1}+\beta^{\prime} b_{1}+$ etc.; $\alpha^{\prime} \sigma_{3}+\beta^{\prime} b_{j}+$ etc. ; the units $a^{\prime}, \beta^{\prime}$, etc., denoting the quantates regarded as constant throughout the first period (say a decennium). At the end of this pretiod a change is mado in the commodities and the units; viz, for the date denoted by $j$ ( $j$ would be 10 if the change were decenmal), and $\boldsymbol{a}^{\prime \prime}, \beta^{\prime \prime}$, etc., are the units used m the second period.

Then we can obtain an imperfect continuaty of tho exchange-values by foming the price-indexes according to the following scheme, viz.: -

$$
\begin{equation*}
\ldots \ldots{ }_{0} I_{1}=\Sigma_{1} / \Sigma_{0} ;{ }_{0} I_{2}=\Sigma_{2} / \Sigma_{0} ; \cdots{ }_{0} I_{j}=\Sigma_{y} / \Sigma_{0} ; \text { etc. } \tag{8}
\end{equation*}
$$

Then if for $\Sigma_{j}$ we form a second sum, using the new unts and denoto this by $\boldsymbol{\Sigma}^{\prime}{ }_{j}$, we shall have-

$$
\begin{equation*}
\cdots{ }_{0} I_{j}=\Sigma_{y} / \Sigma_{0} ;{ }_{j} I_{j+y}=\Sigma_{j+0} / \Sigma_{j} ;{ }_{0} I_{j+j}=\left(\Sigma_{j} / \Sigma_{0}\right)\left(\Sigma_{j+j}^{\prime} / \Sigma_{j}^{\prime}\right) . \tag{9}
\end{equation*}
$$

in which $g$ denotes any year m, the second period; or fully expressed :-

It is obvious from this last expression that any dissmilarity in the aggregate of the units of usage for the two periods does not prejudico the results, directly. Nevertheless it is equally obvious that the results of the second period are not strictly comparable with those of the first period. For the proper relation between any two results should be based on the mean number of units used for tho two datos, and thus would-be as follows:-

Let $a$ denote $\frac{1}{2}\left(\alpha^{\prime}+\alpha^{\prime \prime}\right) ; \beta$ clenote $\frac{1}{3}\left(\beta^{\prime}+\beta^{\prime \prime}\right)$; etc. Then the results for the year say 0 and the year $j+g$ should be

$$
\text { Correct result-- }{ }_{0} l_{j}+a=\frac{a a_{j}+\eta+\beta b_{j}+a+\text { etc. }}{a a_{0}+\beta b_{0}+\text { etc. }}
$$

The tabular results according to formula (10) would, however, differ from this. The measure of this difference we propose now to determme, and wo considor first the case where the changes in the number of units of usage are relatwely small, and whero

[^20]the commodities are the same. In this case we may put $a^{\prime}=a(1-x)$ and $a^{\prime \prime}=(1+x) ; \beta^{\prime}=\beta(1-y)$, and $\beta^{\prime \prime}=\beta(1+y)$, etc., then by interchanging the factors of the numerators the $9 x p r e s s i o n ~(10) ~ m a y ~ b e ~ w r i t t e n--~$
\[

$$
\begin{equation*}
\cdots \frac{a(1+x) a_{j}+a+\beta(1+y) b_{j}+}{a(1-x) a_{0}+\beta(1-y) b_{0}} \frac{a+\text { etc. }}{+1} \cdot \frac{a(1-x) a_{j}+\beta(1-y) b_{j}+\text { cte. }}{a(1-1 \cdot x) a_{j}+\beta(1+y) b_{j}+\text { etc. }} \tag{11}
\end{equation*}
$$

\]

$=\frac{\alpha a_{j+\sigma}+\beta b_{j+0}+\text { etc. }+\left(x \alpha a_{t+g}+y \beta b_{j} \pm a+\text { etc. }\right)}{a a_{0}+\beta b_{0}+\text { etc. }-\left(x a a_{0}+y j b_{0}+\text { etc. }\right)} \times \frac{a a_{j}+\beta b_{j}+\text { etc. }-\left(x a a_{j}+y \beta b_{j}+\text { etc. }\right)}{a a_{j}+\beta b_{j}+\text { etc. }+\left(x a a_{j}+y \beta b_{j}+\text { etc. }\right)}$
If $S$ denote the sum of the quantities outside the brackets, and $s$ the sum of the quantities, within the brackets, then this last expression may be written-

$$
\begin{equation*}
\frac{S_{j}+g+s_{j}+0}{S_{0}-s_{0}} \cdot \frac{S_{j}-s_{j}}{S_{j}+s_{j}}=\frac{S_{j}+g}{s_{j} \div 0} \cdot \frac{\left(1+\frac{s_{j}+g}{S_{j}+a}\right)\left(1-\frac{s_{j}}{S_{j}}\right)}{\left(1-\frac{s_{0}}{S_{0}}\right)\left(1+\frac{s_{j}}{S_{j}^{-}}\right)} \tag{12}
\end{equation*}
$$

Now $S$ is a vory large quantity compared with $s$, therefore $s / S$ is a very small quantity compared with unity, and consequently the right-hand factor in this last equation (with four brackets) must be very nearly unity. It can be seen somewhat more clearly if we put

$$
\begin{equation*}
\ldots \ldots S=\frac{1}{3}\left(S_{0}+S_{j}+S_{j+0}\right) ; s=\frac{1}{3}\left(s_{0}+s_{j}+s_{j+p}\right) \tag{13}
\end{equation*}
$$

and also

$$
\cdot(13 a) \ldots \ldots S_{0}=S(1+\xi) ; S_{j}=S(1+\eta) ; S_{j+v}=S(1+\xi)
$$

and
(13b)

$$
\cdots \cdots s_{0}=s(1+x) ; s_{j}=s(1+\phi) ; s_{j}+a \Rightarrow s(1+\psi)
$$

so that we shall have

$$
(13 c) \ldots \ldots \xi+\eta+\xi=0 ; \text { and } x+\phi+\psi=0 .
$$

The expression (12) then becomes-

$$
\begin{equation*}
\ldots \ldots I_{j+g}=\frac{S_{j+y}}{S_{0}} \cdot \frac{\left\{1+\frac{s}{S}\left(\frac{1+\psi}{1+\xi}\right)\right\}\left\{1-\frac{s}{S}\left(\frac{1+\phi}{1+\eta}\right)\right\}}{\left\{1-\frac{s}{S}\left(\frac{1+\chi}{1+\xi}\right)\right\}\left\{1+\frac{s}{S}\left(\frac{1+\phi}{1+\eta}\right)\right\}} \tag{14}
\end{equation*}
$$

It is obvious that in this expression the whole of the terms denoted by Greek letters are small terms, and are also terms of the same order ; and it is evident, therefore, that unless" prices or weights change very greatly the right-hand factor may'be taken as unity.

It may be pointed out that in actual cases the quantities $S_{0}, S_{j}$, and $\$_{j+j}$ are sensibly identical to the order of, say, several per cent, only ; and $s_{0}, s_{j}$, and $s j+0$ areTusually very small; hence this factor in brackets will in general be so near unity as often to be satisfactory. In other words, the quantities xaa, $y \beta b$ etc., are of a much smaller order than $\alpha a, i b b$, etc., and, entering into the result some with the + and others with the - sign, tend consequently to disappear in the final result.

It may be proper here to note that this last expression shews at once the advantage of the weights being so determined that, for the year on which the basis is changed, the aggregate of expenditures calculated with the two systems of weights shall be identical; for in such a case the values of $a^{\prime}$ and $a^{\prime \prime}, \beta^{\prime}$ and $\beta^{\prime \prime}$, etc., differ on the average the least possible. We may say finally that if the value of the right-hand factor of \{12\} (viz., that containing the four quantities in brackets) is unity then the continuity is satisfactory; if not, then it is unsatisfactory, and in proportion as it differs from unty : this expression or its equivalent (14) affords, therefore, the necessary criterion.

We shall see later that it is desurable that the units for the second period should be so determined that for the year of change $\Sigma_{j}=\Sigma_{j}^{\prime}$. As soon as the relative numbers $a^{\prime \prime}, \beta^{\prime \prime}$, etc., of the various unts have been ascertaned, thas can readily be effected by multiplyng these by an approprate factor, $x$, given by the formula

$$
\begin{equation*}
\kappa=\frac{\Sigma_{j}}{\Sigma_{j}^{\prime}}=\frac{a^{\prime} a_{1}+\beta^{\prime} b_{i}+\text { etc. }}{a^{\prime \prime} a_{j}+\beta^{\prime} b+\text { etc. }} ; \text { that } 18 \times\left(a^{\prime \prime} a_{j}+\text { etc. }\right)=a^{\prime} a_{j}+\text { etc. } \tag{15}
\end{equation*}
$$

Thus we obtan a new set $\left(\alpha^{\prime \prime \prime}=\kappa a^{\prime \prime}, \beta^{\prime \prime \prime}=\kappa \beta^{\prime \prime}\right.$, etc), proportional to those ascertained, viz., $a^{\prime \prime \prime} ; \beta^{\prime \prime \prime}$, etc. When this has been done, then the aggiegate expenditure based on the corrected units for the second period is identical with the aggregate expenditure based on the units for the original period, notwithstending that the system of tunits has been altered. That is, for the year of change, the aggregate expenditure is unaltered, but the distribution among the commodities hes been chànged.
9. Price-Indexes when the Number of Commodities is greatly changed.-We now pass to the consideration of the case where only some of the commodities are common to the two series, and the weights on the occasions compared are very different. In such a case we can continuously trace an exchange-value relation only through the $h$ commodities common to the two groups, and the only thooretically satisfactory comparison is one where the two periods are compared on an identical bass, viz., the arithmetio mean (or more strictly on the geometric mean) of the two series of units. In practical examples it is probable that it is never necessary to use the geometric mean, for in all practical cases the chango of regitten from decennium to decennum can hardly be such as to involve very great differences of weights, or even to involve the alteration of a very large number of commodities. The determnation of relations of $k$ commodities of different weights in the serios of commodities for the two periods will not therefore be unsatisfactory: . In fact, it may be said that in almost every practical example the two mearis (arithmetic and geometris) will give practically adentical results.

The reason of this is that the two means rarely differ very much, as will be seen from the following table, the original unit being $1:-$

| (a) Number of new units $N=1$ | 2 | 3 | 4 | 5 | 9 | 10 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (b) Arithmetic mean , .. 1 | $1 \frac{1}{2}$ | 2 | $2 \frac{1}{2}$ | 3 | 5 | $5 \frac{1}{2}$ | 10 |
| (c) Geometric mean .. 1 | 1.41 | 1.73 | 2.0 | 2.24 | 3 | 3.16 | 4.36 |
| $(b-c) / c$ Percentage of difierence divided by $N$ : 0 | 3.03 | 5.16 | 6.25 | 6.83 | *7.41 | 7.39 | 6.81\% |

* Maximum value.

From this table it is seen that if the new units be $N$ times the precoding units the arithmetic mean wll exceed the geometric mean by never more than 7.4 N per cent. of the latter. Consequently whatever mode we assume for the growth of the unit from one value to another we may take the arsthmetic mean of the units in practical examples.

Reverting to formula preceding, and remembering that the sum in these cases is for the $k$ commodities only, it will still be true that the product of the bracketed quantities in (12) and (14) will be sensibly unity for contiguous decennial periods. In this instance a kind of general continuity can be ostablished even though the regimen is changed (it may be) per salum each decenvium. We proceed to elaborate the question. Whenever the number of commodities has been, changed the question of contmuity can be tested in the following manner, viz. :-

Let $R$ denote the computed aggregate of expenditure on the commodities which appear in the old but not in the new list; $S$ and $S^{\prime}$ denote the aggregato of expenditures on the continued commodities, viz., those appearing in both the old and new lists; $T^{\prime}$ the aggregato of expenditure on those appearmg in the new list only, and let as before the suffixes denote the year to which the expenditure roiers. When the relative values of the units to be used for the new period (that is, for expenditure $S^{\prime}+T^{\prime}$ ) have been found, then these units mast be so corrected, soe formula (15), that the expendit, ... .. the $k$ commodities whose aggregate is $\$$
$S^{\prime}$ shall be identical, whether calculated by the new or by the old units. Then we shall have $\kappa S^{\prime}=S$, and $\kappa T^{\prime}=T$, so that $S$ is identical with either series of units, and $7^{\prime}$ is calculated on the corrected relative umits (the correction making of course no change in their ratio to one another). Then we have by an operation similar to formula (10).
(16) $\ldots{ }_{0} I_{j+j}=\frac{S_{1}+R}{S_{0}+R_{0}} \quad \frac{S_{1+y}+T_{i+j}}{S_{j}+T_{j}}=\frac{S_{j+q}}{S_{0}}$.

$$
\left(\frac{1+\frac{R_{j}}{S_{j}}}{1+\frac{T_{3}}{S_{j}}}\right) \cdot\left(\frac{1+\frac{T_{i+g}}{S_{j+g}}}{1+\frac{R_{0}}{S_{0}}}\right)
$$

Now, sunce $S, / S_{o}$ is continuous under the old system of units, and $S_{j+g} / S_{o}$ is continuous under the new system; $S_{j+p} / S_{0}$ is at least what we have called pseudocontinuous through the entire period, this pseudo-continuty being attained by the correction of the unts, so that the aggregate of expenditure on the $k$ commodities is identical with either system of units.

It can be seen from the above expression that if in introducing new commoditios we take care that the aggregate expenditure on these, with corrected units, exactly equals the expenditure on those omtted at the year of change, we secure this, viz., that the left-hand term in brackets in formula (10) shall be umity, and further that the fractional terms on the right-hand term in brackets shall be of the gatno ordor and also in most cases sensibly equal. For this reason it is emmently desmable that the units be so determined that the whole aggregate of expenditure shall be iclontical wath the new unats as well as the aggregate for the commodities cominon to both groups. Then if the quantaty in the right hand brackets sensibly unity wo can regard the pseudo-continuity as ostablished. In practical examples $g$ should be one, that as the example should apply to the year immedintely following that in which the change in the commodaties and units is made.

Where it is clesired to add a number of commodities such that the expenditure thereon is large as compared with expenditure on those omitted, we rewrite the terms in brackots in (16)

$$
(16 a)^{*} \ldots \ldots{ }_{0} L_{j+0}=\frac{S_{j+!}}{S_{0}}\left(\frac{1+\frac{R_{j}}{S_{j}}}{1+\frac{R_{0}}{S_{0}}}\right) \cdot\left[\begin{array}{c}
1+\frac{T_{j+1}}{S_{j+j}} \\
1+\frac{T_{j}}{S_{j}}
\end{array}\right)
$$

In this $R_{j} / S_{j}$ is a quantity which is ordinarily nearly equal to $R_{0} / S_{0}$, and also $T_{3+f} / S_{j+0} 1 s$ ordinarly comparablo to $T_{j} / S_{j}$. When this condition happens to bo satisfied the continutity may be satisfactory despite the fact that a relatively large addition of cornmodities has been made as compared with those omitted.
10. Effect of Change of Regimen.- When the product of tho factors in (16) and ( 1 (ba) is not unty, then they exhibit approximately the consequence of change of regitnen.

In connection with a discussion on the varmation of the exchange-value of gold the effect of chango of regimen is to be carefully distinguished from mere variation in the magnitude of the units. It ean best be illustrated thus :-

Suppose that, with the same list of commodities for any datum year, and using two series of units, we have equal expendatures, agreeably to the prescription of formula (15), and find with the prices for any other year a difference of expenditure, thus difference measures the effect of change of regimen. To express this otherwise supposs that $I$ and $I^{\prime}$ donote the price-index as deduced with an identical list of commoditios but with two serios of units, of which let us assume I is on an original, and $I^{\prime}$ on a new basis, the expenditures being identical for the datum year. Then we have for $\rho$ the effect of the change of regumen.
(17) $\ldots \ldots \rho=I^{\prime} / I$.

Each year will, of course, give a different value for $\rho$, but if actual results shew that tho variations of $\rho$ are very small, we can regard the (wegghted) mean as furnishing a general measure of the effect of the change. As the distance in time increases from the datum year, the andividual values obviously become of less weight. Hence we may empirically adopt some such formula as
(18) $\ldots \rho_{n}=\frac{\Sigma \frac{\rho_{n}}{n}}{\Sigma \frac{1}{n}}$; or $-\frac{\Sigma \frac{\rho_{n}}{n}}{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}}=\frac{\Sigma \frac{\rho_{n}}{n}}{0.577216+2.3025851 \log _{10} \cdot n+\frac{1}{2 n}-\frac{1}{12 n^{2}}+\frac{1}{120 n^{2}}}$

## Appendix.

if we have the valuos of this factor for successive years 1, 2, 3......n. In gonoral the variations of $p$ will exhibut varations shewng no definte tendency; when this is not the case the progressive change may demand spocal invostigation.
11. Pseudo-Continuity of Price-Indexes with Progressive Change of Regimen.For comparative purposes stretching over long periods of time it would nppear on tho whole desirable to adopt a method, which would be sensibly accurato for short periods of time from the standpont of the exchango-value of gold, and yot novortheless ropresent for long periods the combined effect of change of regmen and alterod exchange-value or purchasing-effieiency of gold, the change of regimen corresponding to varnations in the international usage from period to period. Under such a systom the ratio of price-indexes for distant datos would, strictly speaking, thon cerso to represent changes in the exchange-value of gold but rathor those changos as modified by an alteration of the average regimen. Comparisons from the standpoint of variations in the exchange-value of gold alone would have to be dealt with by special investrgations where necossary. We proceed now to conster the quostion.

The fundamental idea on which a pseudo-continuity can be doveloped is that for the years of change (constituting what we shall call the successive control yoars), the change of units shall be so controlled that the aggregate of expenditure on the $k$ commodities, common to the two groups, shall be jdentical with the two serios of units (formula 15). This gets rid, in probably the most convensont wry, of the difficulty that in general we cannot ascertain the absolute, but only the rolative, number of units used of each commodity.

It will facilitate the explanation to doseribe the method schematically, and the method can best be illustrated as follows :-

$$
\begin{array}{lcc}
\text { Commodities } & \text { Commodities constant } & \text { Commodities being } \\
\text { disappearing. } & \text { to both persods. } & \text { mtroducod. }
\end{array}
$$

1900


I K L M
1910
A BC
D E F G
T K L M

Let 1900 be the last year when commodities say $\mathbf{A}$ to $\mathbf{H}$, aro to be fully includod. It is decided in 1910 to revise the list so that it shalt contain commoditios $\mathbf{D}$ to M , but not $A$ to $C$. In this case 1901 is to be regarded as the change ycar. For this year we must see that the aggregate of expenditure on D to H is equal as required by formula (15) ; and must seo also that, using the old units for A to C, the aggregate of -expenditure is equal (approxmately) to that on $D$ to $M$ working with tho corrected units. When this has been done we decrease the units of A, B, C, yearly by onetenth of the original anount, and merease those of $\mathbf{I}, \mathbf{K}, \mathbf{L}, \mathbf{M}$, yearly one-tenth of their weight for 1900, according to tho following scheane, viz. :-

| Units for Crmmoduties. | Factor corresponding to year. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1900 | 1901. | 1902 |  | 1908 | 1909 | 1910 |
| a, $\beta, \gamma^{*}$ | 18 | 18 | $\frac{8}{10}$ |  | 18 | ${ }_{10}^{10}$ | $\frac{\square}{10}$ |
| ،, к, $\lambda, \mu \dagger$ | $\frac{0}{10}$ | T0 | $7^{2}$ | - . $\cdot$. $\cdot \cdot$ | 10 | $\frac{18}{10}$ | 18 |

* Units of commodities alsappearing. t Units of commodities being Introduced.

Thus in this scheme A, B, C, have entirely disappeared in 1010 , and $T, K, L, M$, have appeared with their full valnes in the same year, while intermediatoly one serios is increasing and the other is diminishing. We also change each of the minits for the commodities D to H one-tenth of their differenco yearly so that the new valaes aro reached in 1910. That is, if $\delta^{\prime}$ denote the corrected weight in 1910, and $\delta$ the weight in 1900, the weight for $1900+n$ years ( $n$ being less than 10), will be

$$
\begin{equation*}
\delta+\frac{n}{10}\left(\delta^{\prime}-\delta\right)=\frac{10-n}{10} \delta+\frac{n}{10} \tag{19}
\end{equation*}
$$

$$
\delta^{\prime}
$$

A very smple numerical illustration will shew the effect of the process, and for this purposo we need take only two commodities which we may suppose to represent the continuous series. These will illustrate the nature of the difference of the two methods. Let us then supposo a regimen of commodities in the first years of a serios to bo th tho ratio of 1 of $A$ to 2 of $B$ : and for the fifth year to be 2.7 of $A$ to 2.4 of B .

We first find by formula ( 15 ), see hereinbefore, that with the prices as at the finab or control year-

$$
I=\frac{2}{3} \quad \text { viz., } \frac{1 @ 4+2 @ 8}{2.7 @ 4+2.4 @ 8}=\frac{20}{30}
$$

Hence the units become 1.8 and 1.6, that is-. .

$$
1 @ 4+2 @ 8=1.8 @ 4+1.6 @ 8
$$

We thius obtain the results in the table hereunder, viz. :-
(i.) for the method of continuously depending upon the original number of units of $k$ commodities, and
(ii.) for the method of changing the units yerrly, respectively-


It is easy to see that the control which ensures the identaty of the final aggregatess (i.) and (it.) for year 5, ensures also that the intermediate values for years 2, 3 and 4 shall substantially agree. Sumilarly, since for the change-year the expenditure on the commodities added $1 s$ to, balance that on those subtracted, we shall get a satis. factory continutity through that year, and thus results which shew thefeffeet mainly of change of price, though modified slightly by change of regimen.
12. Suggested List of Commodities and Scheme of Working.-The following table shews the commoditios included by various authorities in compuling Index-Numbers for different countries. In this tabular statement only commodities which axe common to more than three of the 27 index-numbers have been included; commoditios which are meluded in only one or in ether two or or three of tho mdex-numbers are specified in the notes at the end of the table. Where any commodity is included in more than three of the index-numbers the fact is indicated by a cross (X); in every case where more than one grade or quality of any commodity is included the small number shewn in brackets after the cross. specifies the number of grades or qualities.
Take in Table.

Commodities included in

| Commodity． | Great Britam． |  |  |  |  |  |  | Germany |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 鰝号 |  |  |  |  |  |  |  |  |  |  |
| No．ofCommodities | 39 | 22 | 20 | 22 | 39 | 39 | 45 | 48 | 47 | 22 | 47 | 114 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Hides ． | X |  |  |  | X | $\mathrm{X}(2)$ | X | X | $\ldots$ | $\cdots$ |  | x |
| Leather ．－ | $\because$ | $\frac{\mathrm{X}}{\mathrm{x}}$ | X | X | X | ${ }^{x}$ | $\cdots$ | $\mathbf{x}$ |  | ． | r | X |
|  | $\because$ | $\frac{\mathrm{x}}{\mathrm{X}(2)}$ | $\underset{\mathrm{x}}{ }$ | $\frac{X}{X(2)}$ | $\underset{\text { X }}{\text {（3）}}$ | $\underline{\mathbf{X}} \mathbf{}$ | $\dot{X}$ | X | $\underset{\sim}{x}$ | x | $\underset{\mathbf{x}}{ }$ | X |
| ＂Cloth | $\ldots$ | ${ }^{\mathbf{x}}$ | ． | $\frac{\mathrm{X}}{1}$ | － |  | X | ．． | $\dot{\sim}$ |  | － | X（3） |
| Flax ${ }_{\text {\＃}}$ | $\dot{\mathbf{x}}$ | ${ }_{\text {N }}$ | ¢ | $\frac{\mathrm{X}}{\mathbf{X}}$ | x | $\dot{\mathbf{x}}(2)$ | $\dot{\mathbf{x}}$ | $\stackrel{+}{\mathbf{x}}$ | N | $\cdots$ | X | ＊ |
| Homp ．．．． | $\cdots$ | $\mathbf{X}$ | X | $\boldsymbol{X}$ | X | X 2 （2） |  | X | X | ． | X | X |
| Jute $\quad . \quad \begin{aligned} & \text { Jinen }\end{aligned}$ | $\because$ | ． |  | $\cdots$ |  |  | X |  | x | $\cdots$ | x | $\dot{\mathbf{x}}(\underline{2})$ |
| Silk，Raw $\quad$. | $\because$ | x | $\stackrel{\mathrm{x}}{\mathrm{x}}$ ． | $\dot{\chi}$ | $\underline{\mathrm{x}}$ | $\dot{x}$ | $\dot{x}$ |  | $\underline{x}$ | $\ddot{\mathrm{x}}$ | X | $\times$ |
| Woollen Yarn ． | X |  |  | X | X | $\mathbf{X}(3)$ | X 2 ） | $\mathbf{x}$ | x | ．． | X | 文（2） |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Barley Being $\cdots \quad \cdots$ | $\frac{\mathrm{x}}{\mathrm{X}}$ | $\cdots$ | $\because$ | $\because$ | X | x | $\mathrm{X}(2)$ | x | 入 | x | $x$ | － |
| Clover ：．$\because$ |  | $\because$ | $\because$ | $\because$ | $\mathbf{X}$ | $\ldots$ | $\ldots$ | x | X | $\cdots$ | X | $\overrightarrow{\mathrm{X}}$ |
| Hay ．．．． | $\mathbf{X}$ | ． | ．． | ． | X |  |  |  | ． |  |  |  |
| Linseed | $\ldots$ | $\cdots$ | $\cdots$ | $\cdots$ | ． | X | x | $\cdots$ | $\ldots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| Mraize ${ }_{\text {Oats }}$ | $\ddot{\mathrm{x}}$ | $\cdots$ | $\cdots$ | $\because$ | X | $\underset{\sim}{x}$ | $\underset{\text { X }}{ }$ | x | x | $\ddot{\mathrm{x}}$ | X | x |
| Peas ： | X | $\ldots$ | ． |  | $x$ |  |  |  |  |  |  | X |
| Rape seed ． | ． | $\cdots$ | ． | $\because$ |  |  |  | X | $\ddot{\text { x }}$ |  | $\dot{\chi}$ | － |
| Rice ．${ }^{\text {Rye }}$ | $\mathbf{x}$ | $\cdots$ | $\cdots$ | $\cdots$ |  | X | X | X | － | ＋ | स | स |
| Straw ．． |  | $\cdots$ | $\because$ | $\because$ | $\stackrel{1}{x}$ |  | $\cdots$ |  |  |  |  |  |
| Wheat | X | x | X | X | X | $\underline{\mathrm{X}}$（2） | x＇（2） | $\dot{\mathbf{x}}$ | $\ddot{\mathrm{x}}$ | $\dot{X}$ | $\dot{x}$ | $\dot{\chi}$ |
| Bacon | ． |  | $\cdots$ | ．． |  |  | x |  |  |  |  |  |
| Butter－ | $\because$ | $\cdots$ | $\because$ | $\because$ | $\ddot{\mathbf{x}}$ | X | $\ldots$ | $\dot{x}$ | x | $\because$ | \％ | $\ddot{x}$ |
| Cheese ．． | X | $\cdots$ | $\because$ | ． | ． | ．． |  | X | ． | $\cdots$ | ． | $x$ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| ${ }_{\text {Almonds }}$ | X | ＊ | $\cdots$ | ． | $\cdots$ | $\ldots$ |  | X | X |  | X | X |
| Coffee $\because . \quad \therefore$ | $\because$ | $\ddot{\mathrm{X}}$ | $\dot{\mathbf{x}}$ | $\dot{x}$ | $\because$ | $\underline{\mathrm{X}}$（2） | $\underset{\chi}{ }$ | X ${ }^{\text {（3）}}$ | ＊ | स | N | Y |
| Currants | $\cdots$ | $\cdots$ | $\cdots$ | ． | $\cdots$ | ＋ | ．． | $\boldsymbol{X}$ | X | ． | X | X |
| Flour－ | ． | $\cdots$ | $\because$ | $\cdots$ | $\cdots$ | $\mathbf{X}$ | x | $\boldsymbol{\lambda}$ | － | $\cdots$ | ． | ＋ |
| Imard $\because \quad \because$ | $\because$ | $\cdots$ | $\cdots$ | $\because$ | $\because$ | $\because$ | $\cdots$ | $\cdots$ | X | $\because$ | $\ddot{\mathrm{X}}$ | X |
| Malt ．．．． | X | $\cdots$ | $\cdots$ | $\cdots$ |  |  | ． |  |  |  |  | X |
| Pepper ．．． | ． | ． | $\cdots$ | － | X | x | $\cdots$ | $\overline{\mathrm{x}}$ | $x$ | $x$ | $\mathbf{x}$ | N |
| Rasins．．$\quad$. | $\ddot{\mathrm{x}}$ | $\cdots$ | $\because$ | $\ldots$ | $\because$ | $\boldsymbol{X}$ | $\cdots$ | $\ddot{\mathrm{x}}$ | $\stackrel{\rightharpoonup}{\mathbf{X}}$ | $\because$ | $\dot{\text { x }}$ | X |
| Rye Fiour |  | $\cdots$ | $\cdots$ | $\cdots$ | ． | ． | $\cdots$ |  | $\ldots$ | $\cdots$ | ． | F |
| Salt Spirits | X | $\cdots$ | $\cdots$ | $\cdots$ | X |  |  |  | ． | $\cdots$ | $\cdots$ | x |
| Sugarits $\because=\sim$ | ．． | X | x | X | X | $\dot{\text { x }}$（3） | X | $\underline{\mathrm{X}}(2)$ |  |  | $\dot{x}$ | $\frac{\mathrm{X}}{\mathrm{X}}(2)$ |
| ＇＇ear ．．． | $\cdots$ | X | X | X | X | X（2） | $\mathbf{X}$ | ${ }^{\mathbf{X}}$ | $\mathbf{X}$ | X | X | X |
| Tobacco | ．． | X | X | X | ．． | ．． | X | x |  |  |  | X |
| $\underset{\text { Meef }}{\text { Meat，}}$ | $\cdots$ | X | X | X | x | X（2） |  | N |  |  |  |  |
| Mutton ．．．． | $\because$ | x | ．． | ． | X | $\mathrm{X}(2)$ | X | $x$ |  | $\because$ | $\because$ | $\frac{\mathrm{x}}{\mathbf{R}}$ |
| Pork ．－ |  | $\cdots$ | $\cdots$ | $\ldots$ | X | X | $\cdots$ | X | $\cdots$ | ． | $\cdots$ | X |
| Fish ． | $\frac{\mathrm{x}}{\mathrm{x}}$ | ． | $\cdots$ | $\cdots$ | ．． | $\cdots$ | $x$ | X | $\dot{\mathrm{x}}$ | $\cdots$ | － | K（2） |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | $\because$ | x | x | x | x | X $(2)$ | X | $\cdots$ | $\because$ | $\cdots$ | $\cdots$ | $\cdots$ |
| Miscellaneous．$\quad \mathrm{X}$（ X （ X |  |  |  |  |  |  |  |  |  |  |  |  |
| Canutchouc ．． |  |  |  |  |  |  | X |  |  |  |  | X |
| Indigo ．．．．． | $\cdots$ | $\frac{\mathrm{X}}{\mathrm{X}}$ | X | $\underline{x}$ | X | x |  | $x$ | x | $x$ | $x$ | X |
| Oils ${ }^{\text {Saltpetre }}$ | $\because$ | X | $\boldsymbol{X}$ | $\underline{ }$ | $\mathbf{X}(2)$ | $\mathbf{X}$（3） | $\mathrm{X}(2)$ | $\frac{\mathrm{X}}{\mathbf{X}}$ | ${ }^{\mathbf{X}(4)}$ | $\frac{\lambda}{\lambda}$ | ${ }^{2}(5)$ | N（5） |
| Soda | $\cdots$ | ． | $\cdots$ | ． | $\cdots$ | $\dot{\mathrm{x}}$ | $\cdots$ | $\mathbf{X}$ | － | J | ． | X |

various Investigations.


In the following notes particulars are given ragarding commodities included in each of the above index-numbers, but excluded from the preceding table for the reason already stated.

Fleetwood.-Cloth, Shoes, Ploughs, Carts, Land, Horsos, Cattle, Mules, Swino, Goats, Fowls, Rabbits, Pıgeons, Wine, Ale, Beer, Spice, Wax, Figs, Charcoal.

## Sevons.-Tin Plates, Logwood.

Sauerbeck.--Petroleum, Nitrate of Soda.
Board of Trade,-Wine, Cetton Seed, Paraffin, Petroleum.
Laspeyres.--Logwood, Calfskin, Rags, Tar, Wine.
Pausche.-Mahogany, Calfskins, Bristles, Horse-hair, Wax, Quiçsılver.
Conrad.-Mahogany, Hops, Calfskitis, Bristles, Horse-hair, Wax, Qucksilvor.
Soetbeer-Buckwheat, Oilcake, Veal, Calfskms, Horse-har, Bristlos, Bedfeathers, Bones, Buffalo Horns, Glue, Dried Prunes, Wine, Champagne, Allspico, Cassia Bark, Sago, Cochineal, Logwood, Rosewood, Mahogany, Ratters, Ivory, Quicksilver, Sulphur, Lime, Cement, Corclage, Rags, Gunno, Gun-olastic, Resin, Pearl Ash, Pitch, Potash, Candles, Tar, Wax, Sewing Thread, Bottlds, Sailcloth, Woollen Cloth, Flannels, Worsted, Carpets.

- Prusstan Govermment-Lentils, Veal. 37 articles are given but only 15 are specified.

Hooker (Germany), Cattle, Calves, Pigs (2), Hops, Potrolourn.
Palgrave (France).—Ot-seed, Sılk Stufis, Gloves.
Pallner (France).-Becves, Calves, Cows, Hogs, Sesamun, Lambskins, Kidskins, Silk Goods (2), Merinos, Blankets, Carpets, Tapestry, Glovos.

Hooker (France).-Cattle, Calves, Pigs, Wine, Nitrate of Soda.
Italy (Government).-Wine.
Walras (Switzerland).-Bread (2), Veal, Frewood (2).

- Athinson (India).-Mace, Millet Corn, Pulse, Fajra, other Grains, Gingor, Opium, Croton, Castor Oil, Dye, Bone Manure, Jute Goods, Silk Coods, Shelre.

Aldrich (U.S.A.).-Ship Bread (3), Boston Crackors (2), Oyster Crackors, Ship Biscuits, Soda Cracker's, Dried Apples, Com Meal, Ham, Lamb, Molassos (2), Nutmegs, Cornstarch (2), Blankets (2), Broadcloths (2), Calico, Carpets (3), Cassimeres (4), Checks, Horse Blankets, Print Cloths (2), Shawls, Sheetings, Shirlongs, Tickmg, Candles, Matches, Anvils, Butts, Door Knobs, Lead Shot, Locks (2), Meat Cutters, Cut Nails, Pocket Knives (25), Qucksilver, Rope (3), Saws (4), Scythes, Shovels, Wood Screws, Carbonate of Lead, Cement, Doors, Limo, Oxide of Zinc, Plate Glass (6), Putty, Tar, Turpentine, Wholow Glass, Alum, Potash, Vitriol, Brimstone, Calomel, Copperas, Flax Seed, Glycerine, Mercury, Muriatic Acid, Opium, Quinıne, Soda Ash, Sugar of Lead (2), Sulphuric Acid, Furniture (3), Glassware (5), Pails (3), Tubs (4), Powder (2), Soap, Starch.

閪 Bureau of Lator (U.S.A.)- Conned Corn, Canned Pers, Canned Tomatoes, Dried Apples, Prunes, Glucose, Corn Meal (2), Molasses; Fresh Vegatables, Onions, Broadeloth, Drill, Gingham, Horse Bhankets, Hosiery, Overcontings, Sheotings, Shirtings, Tickings, Underwear, Sicilian Cloth, Cashmere, Poplar, Janana. Worsted and Worsted Yarn, Candles, Matches, Augers, Axes, Barb Wire, Butts, Chisols, Coppor Wire, Steel Door Knolss, Filos, Hammers, Lead Pipe, Loeks, Cut Narls. Wre Nails, Planes, Saws (2), Shovels, Steel Bıllets, Steel Rails. Steel Sheots, Trowels, Vises, Wood Screws, Whate Lead, Cement (2), Doors, Limo, Oxido of Zine, Plate Glass, Putty, Resin, Shingles, Purpentine, Window Glass, Alum, Brimstono, Glycerme, Muriatic Acid, Opium, Qumine, Stalphuric Acid, Earthenwrore Piateg, Cups and Saucers, Bed Sets, Chaurs (2), Tables, Glassware (3), Gutlery, Woodenwaro, Cotton Seed, Meal, Newspaper, Wrapping Paper, Rope, Soap. Cattle, Fowls, Horses, Mules, Şwme, Bread, Blankets, Carpets, Shoes and Boots (4), Quicksilver.

Coats (Caneda). Bran, Shorts, Turkeys, Chocolate, Chean of Tartar, Fresh Fruit (6), Honey, Maple Sugar, Oatmeal, Molasses, Tapioca, Vegetables (3), Canned Vegetables (3), Vinegar, Brass, Solder, Anvils, Axes, Grindstones, Hammers, Horseshoes, Mallets, Pıoks, Screws, Solderng Lrons, Vices, Coke, Carbide of Calcium, Matches, Hangas, Wure Nails, Cut Nails, Plaster of Paris, Sash Weights, Soll Pipe, Ware Cloth, Wire Fencing, Paints, Glass, Benzine, Glue, Bonled Oil, Putty, Paris Greon, Shellac, Turpontme, Varnish, White Lead, Chairs, Tables, Sideboards, Bod Suites, Beds, Tumblers, Cups and Saucers, Toilot Sets, Dinner Sets, Knives, E.J. Jnives and Forks, Wood Pails, Wood Tubs, Brooms, Alum, Bleaching Fowder, Borax, Carbolic Aerd, Caustic Sods, Copperas, Glycerine, Muriatie Acid, Opium, Quitine, Socla Ash, Sulphuric Acid, Furs (4), Binder Twine, Ropo, Soap, Cattle, Beer, Shoes and Boots (3);

Australia (Wholesale),-Branbags, Cornsacks, Woolpacks, Leather (3), Bran, Pollard, Oatmoal, Ham, Honoy, Macaroni, Sago, Mustard, Stareh, Blúe, Matches, Candles, Kerosene, Veal, Lamb, Cement, White Lead, Cream of Tartar, Sulphur.

Australia (Retail).-Bread, Sago, Jan, Oatmeal, Stareh, Blue, Candles, Soap,
Hans, Ham. Onions, Ham.

In addition to the alithoritios mentioned in the above table, investigations have also boon made in the following countries, but detalss as to the commodities included therein are not available :-

| Country. | Name of Authority. | Years. | No. of Articles. |
| :---: | :---: | :---: | :---: |
| Great Britain- | Race Vauphan Evelyn Mulhal | $\begin{gathered} 1675 \\ 1798 \\ 1854-1884 \end{gathered}$ | 50 |
| Germany (Hamburg)- | Kral Heznz Schmelz | $\begin{aligned} & 1845-1884 \\ & 1850-1891 \\ & 1890-1910 \end{aligned}$ | $\begin{array}{r} 265 \\ 180 \\ 29 \end{array}$ |
| $\underset{1}{\text { France- }}$ | D'Avenal <br> De Fovrlle <br> Reforme Economique | $\begin{aligned} & 1200-1898 \\ & 1847-1880 \end{aligned}$ |  |
| Belgium- | Waxweiler | 1890-1910 |  |
| U.S. A- | Burchard Prollaner | $\begin{aligned} & 1825-1884 \\ & 1890-1899 \end{aligned}$ | 68 to 90 articles 90 articles |
| New Zealard- | - McIlwrath | 1861-1910 | 33 to 45 artjeles |

An examination of the above statement clearly shews the great diversity in practico which existod in tho solection of commodities in order to obtain the price data for the computation of Index-numbers. It may, be seen that not one of the 67 commodities specified is common top all the Index-numbers. Several commodities in ordinary use, such as coal, iron, cotton, wool, whoat, butter, etc., etc., are, however, common to the majority of the groups.

Applying the principlos which have already been laid down in this Appendix for the selection of a group of commodities for the purpose of international comparisons the fellowing list has been compiled. Suggested " mass-untts" (indicating relative consumption of each commodity in the corresponding unit of measurement) are also shewn in the following statement. These " mass-units" are based almostentirely on tho Australian consumption, and are therefore suggested tentatively; they will probably require some amendment for international purposes.

## Proposed List of Commodities Suitable for Comparative Index-Numbers for the Western Nations with Mass Units.



Summary of Conclusions.-The conclusion of the whole matter divides itself into two heads, viz., (i.) that which concerns the list of commodities, the number of unts to be taken, and changes in this list ; and (it). that which concerns the technique of computing the price-index.

Regarding the first we may say as follows, viz. :-
I. (i.) The Itst should contain (a) commodities easly identifiable as to character and quality; ( $b$ ) commodities for which there are world markets. Commodities for which only a local market exists should constitute a separate list for local purposes.
(ii.) The number of units taken should represent the average usage among all the nations included in the comparison.
(iit.) The number of cominodities and the unts assigned should be subject to decennial revision.
(iv.) During each decennium, the series of units and commodities used must necessarily be those ascertained for the preceding decenmum.
(v.) At the close of each deconmmon it is desirable that the price-indexes found for th should be rovised on the introduction of the next decennial Inst of commodities, and the units of usago assigned to them.
(vi.) In order that the price-indexes, while substantally accurate from the standpoint of gold exchange-value, shall yot represent the actual usage of mankind in respect to commoditics, its basis, owing to change of normal regimen, should be subject to continuous modifications.
(vii.) This is practically secured by varying the regimen anits of commodities yearly ono-tenth of the decemmal chfference, the control of the number of units assigned being properly attended to.
(vii.) Subdivisions of the fist of commodities should be so made that the atems withn a subdivisuon are homogeneous with respect to the ratio of the value of the raw material to the value of the labour in the finished procluct.
(ix.) There can be no really perfect continuty between the price-indexes for poriods characterised by different regimens.
(x.) Since economo inquiries of an exact character must take aecount of variations in the relativo usage of commodittes, comparisons between widely different periods must take account not only of variations in the exchange-valuo of gold, but also in average regmen.

In regard to technique, the common-sense method of adopting, for the purposes of comparison, a series of umits of definite cointnodities and finding the aggregate of expenditure according to these, is unquestionably the best method of tracing the variations in the exchange-value of gold against commodities. The matter may be summed up as iollows:-
II. (t.) For initial comparısons, the experience of each decennıun will furmish the tunits that are used for the following decenniums.
(ii.) The method of firding the ratio of aggregate expenditures is not only the stmplest but the best.
(iii.) Price-ratios are not satisfactory anless the weighted geometric mean be found, and using for the weights assigned the mean expenditure for any two periods. The method then becomes sensibly identical with the aggregate expenditure method, but the arithmetical work involved is prohibitive, and the method is not suited for continuous records.
(iv.) Although the apparent generality of the price-ration method is not wholly an ilhasion, it practically has no advantages whatever over the aggregate of expenditure method, the latter being arithmetically very simple.
(v.) With the aggregate of expenditure method, the influence of any uncertainty in the series of commodities or $i n$ their prices, on the priceindex deduced, can more readily be seen than with the price-ratio method.
(vi.) The establishment of an international series of commodities would have for its immediate object the comparison of the exchange-value of the gold-untt in each nation on the basis of a common average regimen.
(vii.) This may not be the best system of units for the nation itself.
(viii.) Each nation may find it necessary, therefore, to have also its own list, and its own units, and to deduce price-indexes reprosenteng the variation of the exchange-value of gold so far as the nation itself is concerned in its internal relations.
(ix.) In general the fluetuations on the two bases will not be quite identical, the difference being due to what may be called change of regimen.
(x.) Experience may, however, shew that the relation between the two can be readily determined, or is a negligible quantity, so that ultimately one itst may suffice.

Regarding general matters the following may be said :-
III. (i.) It may, on first consideration, appear unsatisfactory that through long intervais of time the same class of commodities cannot be utilused for determining absolutely variations in the exchange-value of gold. If; however, the method involving slow variations of regmen be followod, there is no strong objection to the method indicated in this papor.
(ii.) Per contra, it is to be preferred, since it applies to the exsting regimen at all points of time, at least when corrected-as indieated by contimuously varying the regimen.
(iii.) By these methods a satisfactory kind of continuity can be secured, which although only a pseudo-continuty as regards the exchange.value of gold, is nevertiseless a real continuity as regards tho usage of gold in relation to all other commodities on the list.
(iv.) It is therofore of much greater value then would be furnished by pricoindexes based, if it were possible-wheh it is not-on a continued use of the commodities of the past as the basis of determination.
(v.) The method of a slowly changing " commodity unit," though establishing theoretically only approximate values, nevortheless yiolds resulte which more truly represent the aggregate of the facts, than does the method of absolute comparisons besed upon the same number of units and the same list of commodities.
(vi.) Speciai investigations may nevertheless be considered necessary between any two years for any definite series of commodities, and any definite number of units in connection therewith.


[^0]:    * This cannot, of course, be strictly calculated tor reagons which will be clear on referring to Appendixes VIII. and IX.

[^1]:    *See "On Prices of Commodities and the Precious Metals,"'A. Sanerbeck. "Journal of the Royal Statistical Soclety, London, Vol 49 (1868), p 587.
    *See "An Introduction to the Study of Prices." W T. Leyton, M.A., London, 1918.

[^2]:    * See " The Standard of Velue." Sir David Barisomr, K.C.S L., K.C.M.G., London, 19is.

[^3]:    "See "The Purcbasting Power of Money." Prof. Irving Fisher. New York, 1911
    4 See "Report of Commission on Cost of Living in New Zealand." Wellington, 1912, p. xxyv.
    $\ddagger$ The "Equation of Exchange" may be exprassed mathematically fs follows -
    If if represent the quantity of actual currency money, and $\vec{V}$ its velocity of circulation, $M_{3}$ the cuantity of credit money and $F_{2}$ Its velocity of circulation; also if the average prices of the various coinmodities sold during the period under reyiew be $P_{1}, P_{2}, P_{3}$, etc., and the corresponding quantities sold be $9^{2} \cdot Q^{2}, O^{3}$, etc respectively, then
    $M V+\mathbf{M} \boldsymbol{I}_{1}=\Sigma(P Q)$,
    which may be written
    $M V+M_{1} V_{1}=P T$, where $P$ is a weighted average of all the $P^{\prime} s$, and $T$ is the sum of all the G's. $P$ then represents 12 ove magnitude the level of prices, and $T$ represents in one marnitude the volume of trade.

    $$
    \text { The price level } P=\frac{M V+M M_{1} V_{2}}{T}
    $$

[^4]:    "See "An Introduction to the Atudy of Prices," W. T. Layton. M.A, London, 1911.

[^5]:    *See "Cazsees of the Riso of Prices" by J. A. Holoson. "The Contemporary Revjew," No. 562, Octoler 1912.

[^6]:    * First 9 months.

[^7]:    * Furst 9 months.

[^8]:    * Average prices for frst 9 months only.

[^9]:    * Average prices for arst 9 months only.

[^10]:    * Average prices for first 9 months only.

[^11]:    

[^12]:    * Forasmuch as the money-unit constitutes a unique common measure of exchange-values.

[^13]:    * The consumption per head per annum Is about 82 loaves of bread, 3 lbs . of tea, and 16 quarts of milk.
    $\dagger$ Here it may be mentioned that computed from the geometric mean of the price-ratios, weighted according to the arithmetic mean of the weights, we ghould obtain 109.53. See next section.
    \% This method is wholly unsatisfactory.

[^14]:    * It is of course evklent that if this can be done it is also posslible to work with the relative units used of the various commoditaes; thus formula ( 2 ) is more convenient. It is also to be preferred in every wity as will hereafter be shewn.
    $\dagger$ As given by formula (2) since the units are iflentical.

[^15]:    * By formula (2), viz, the ratio of the aggregate expenditures, we get 109.47; using arithmetic-mean weights and formuta (11) we get 10958. ,
    $\dagger$ It js shewn hereimafter that formulec (11) and \{2) are sensibly tlentical when the weighto and units are properly determined.

[^16]:    * The question of variation of regimen I have considered elsempers, but not hersin.
    $t$ This bis been done elsewhere.

[^17]:    * For so lons as the unintelligence and bad-will of maukind necessitetes so wasteful a procedure, the commodity pold (and siver) may be regarded as in some winy the real basis of theidon of woney, and this notwithstading the fact thet the use of the precious metals will probably bo sreatly limited or way even cease when international obligations are certan to be bonoured, or when an international credit system is suthelently assured: a consummation which donktioss will tend to be reached in proportion as the jeopardy of war is duminished So long as we bear in mind that we are thoking of money in geteral, rather than actual gold, we may use the oxpression "purchasing-efficteuey or exchange-value of told" to represent that reciprocal of the relation between the commodity golk and any other conmodity, which is expressed as price in this viow "price" is looked upon as definmg the instantalieous potentiality ot exchange, by the artifice of a supposed real commodity, vz , gold.

[^18]:    * Contrast such forms of sfeel manufacture as heavy spíngs for ralway fittings, with waiclin springs or contrast say the production of heavy and cliffon sitks. $\psi$ For cxampte with such commodities as watch-springs, in which the value of the raxv material is wholdy negligible, the resultang price dopends practacally wholly upon the cost of labout directly or midirectly.

[^19]:    * To revert to a former illustration, the value of a watch-spring may be said to be due wholly to the cost of labour requred to produce it, and it stands, therefore, in a very different economic prosition to, say a large madsimple casting, the raw material bcing pis-iron, because for the production of the latter the elemont of cost of labour enters relatively to a much less extent. And even if in the last analysis it conld bo assumed that the onginal raw material is without vaiue until labour us expended thereon, which is not always true, the fact still remains that we shall do well in any classincation to have regard to the value of labour in production, as compared with the value of the raw material

[^20]:    *This, as has been shewn, gives in general almost the same results as the geometric mean.

